

①

$$I = \int_{-1}^1 f(x) dx \approx a_0 f(-1) + a_1 f(x_1) + a_2 f(x_2) + a_3 f(1)$$

$$\pi(x) = (x - x_1)(x - x_2) = x^2 - 5x + P$$

$$\pi^*(x) = (x+1)(x-1)$$

$$\int_{-1}^1 \pi(x) \cdot \pi^*(x) w(x) dx = 0$$

$$w(x) = ax + b$$

$$\Rightarrow \int_{-1}^1 (x^2 - 5x + P)(x^2 - 1)(ax + b) dx = 0$$

$$\int_{-1}^1 (x^4 - 5x^3 + (P-1)x^2 + 5x - P)(ax + b) dx = 0$$

$$\int_{-1}^1 [ax^5 + (b - 5a)x^4 + (aP - a - 5b)x^3 + (5a + Pb - b)x^2 + (5b - Pa)x - Pb] dx = 0$$

$$2 \frac{b - 5a}{5} + 2 \frac{5a + Pb - b}{3} - 2Pb = 0$$

$$3b - 3a5 + 5a5 + 5bP - 5b - 15Pb = 0$$

$$a\{25 + b(3 + 5P - 5 - 15P)\} = 0 \Rightarrow$$

$$\Rightarrow \begin{cases} 25 = 0 \\ -2 - 10P = 0 \end{cases} \Rightarrow \begin{cases} S = 0 \\ P = -\frac{1}{5} \end{cases}$$

$$x^2 - \frac{1}{5} = 0 \Rightarrow x_{1,2} = \pm \frac{\sqrt{5}}{5}$$

$$\begin{cases} a_0 + a_1 + a_2 + a_3 = \int_{-1}^1 dx = 2 \\ -a_0 + \frac{a_1\sqrt{5}}{5} + \frac{a_2\sqrt{5}}{5} + a_3 = \int_{-1}^1 x dx = 0 \\ a_0 + \frac{1}{5}a_1 + a_2\frac{1}{5} + a_3 = \int_{-1}^1 x^2 dx = \frac{2}{3} \\ -a_0 + \frac{a_1\sqrt{5}}{25} + a_2\frac{\sqrt{5}}{25} + a_3 = \int_{-1}^1 x^3 dx = 0 \end{cases}$$

$$\begin{cases} \frac{4}{5}(a_1 + a_2) = \frac{4}{3} \\ \frac{4\sqrt{5}}{25}(-a_1 + a_2) = 0 \Rightarrow a_1 = a_2 \end{cases}$$

$$a_1 + a_2 = \frac{5}{3}$$

$$2a_1 = \frac{5}{3} \Rightarrow a_1 = \frac{5}{6}$$

$$\boxed{a_1 = a_2 = \frac{5}{6}}$$

$$\begin{cases} a_0 + a_3 = \frac{1}{3} \\ -a_0 + a_3 = 0 \end{cases}$$

$$\Rightarrow a_0 = a_3$$

$$\boxed{a_0 = a_3 = \frac{1}{6}}$$

Deci

$$I = \frac{1}{6}f(-1) + \frac{5}{6}f\left(-\frac{\sqrt{5}}{5}\right) + \frac{5}{6}f\left(\frac{\sqrt{5}}{5}\right) + \frac{1}{6}f(1)$$

② Să se calculeze integralele

$$a) \int_a^b \frac{f(t)}{\sqrt{(b-t)(t-a)}} dt = I_1$$

$$b) \int_a^b \sqrt{(b-t)(t-a)} f(t) dt$$

$$t = \frac{b-a}{2}x + \frac{b+a}{2}$$

$$dt = \frac{b-a}{2} dx$$

$$I_1 = \int_{-1}^1 \frac{f\left(\frac{b-a}{2}x + \frac{b+a}{2}\right)}{\sqrt{\frac{b-a}{2}(x-1)\frac{b-a}{2}(x+1)}} \frac{b-a}{2} dx =$$

$$= \int_{-1}^1 \frac{f\left(\frac{b-a}{2}x + \frac{b+a}{2}\right)}{\sqrt{1-x^2}} dx = \int_{-1}^1 \frac{g(x)}{\sqrt{1-x^2}} dx$$

gr. de valab. 3

$$I_1 \approx a_0 g(x_0) + a_1 g(x_1) + a_2 g(x_2)$$

$$a_0 = a_1 = a_2 = \frac{\pi}{3}$$

$$\begin{cases} x_0 = -\frac{\sqrt{3}}{2} \\ x_1 = 0 \\ x_2 = \frac{\sqrt{3}}{2} \end{cases}$$

$$(3) \quad I = [x_0, x_0 + a]$$

$$f: I \times \mathbb{R} \longrightarrow \mathbb{R}$$

$$y' = f(x, y)$$

$$y(x) = \lambda + \int_{x_0}^x f(t, y(t)) dt$$

$$\begin{cases} y_0 = \lambda \\ y_{i+1} = y_i + h \cdot f(x_i, y_i) \end{cases}$$

$$x_i = x_0 + i \cdot \frac{a}{n} \quad i=0:n$$

$$y_0, y_1, y_2, \dots, y_n$$

$$\begin{cases} y_0 = y_i \\ y_{ji} = y_i + h \sum_{l=0}^{j-1} \underbrace{k_{jl}}_{?} f(x_{jl}, y_{jl}) \end{cases}$$

$$y(x_i) = y_i$$

$$x_i = x_0 + i h, \quad i=0:n$$

$$y' = f(x, y)$$

$$y_k = y_{k-2} + \frac{h}{3} (a_2 f_{k-2} + a_1 f_{k-1} + a_0 f_k)$$

$$y'(x_i) = f_i$$

$$y^3$$

$$f = 3x^2$$

- il faut trouver p et q en exigeant
 $x_i = ih$ (un scalaire ne modifie pas la prop. avec f)
 $x_i = i$

$$y = 1 \quad f = 0$$

$$1 = 1 + \frac{1}{3} (0 + 0 + 0)$$

$$y = x \quad f = 1$$

$$k = k - 2 + \frac{1}{3} (a_2 + a_1 + a_0)$$

$$\boxed{a_2 + a_1 + a_0 = 6}$$

$$y = x^2 \quad f = 2x$$

$$k^2 = (k-2)^2 + \frac{1}{3} (2a_2(k-2) + 2a_1(k-1) + 2a_0k)$$

$$k=0 \Rightarrow 0 = 4 + \frac{1}{3} (-4a_2 - 2a_1)$$

$$\boxed{a_1 + 2a_2 = 6}$$

$$y = x^3 \quad f = 3x^2$$

$$k^3 = (k-2)^3 + \frac{1}{3} (3a_2(k-2)^2 + 3a_1(k-1)^2 + 3a_0k^2)$$

$$k=0 \Rightarrow 0 = -8 + \frac{1}{3} (12a_2 + 3a_1)$$

$$3a_1 + 12a_2 = 24$$

$$\boxed{a_1 + 4a_2 = 8}$$

$$\begin{cases} a_0 + a_1 + a_2 = 6 \\ a_1 + 2a_2 = 6 \\ a_1 + 4a_2 = 8 \end{cases} \Rightarrow \begin{cases} a_2 = 1 \\ a_1 = 4 \\ a_0 = 1 \end{cases}$$

$$y_k = y_{k-2} + \frac{h}{3} (f_{k-2} + 4f_{k-1} + f_k)$$

gr. de volob = ?

$$y = x^4 \quad f = 4x^3 \Rightarrow k^4 = (k-2)^4 + \frac{1}{3} (4(k-2)^3 + 16(k-1)^3 + 4k^3)$$

$$0 = 16 + \frac{1}{3} (4(-8) - 16)$$

$$0 = 0$$

$$y = x^5 \quad f = 5x^4 \Rightarrow k^5 = (k-2)^5 + \frac{1}{3} (5(k-2)^4 + 20(k-1)^4 + 5k^4)$$

$$0 \neq -32 + \frac{1}{3} (90 + 20)$$

$$\Rightarrow \text{gr. de volob} = 4$$

④

$$y' = f(x, y)$$

$$y(x_0) = y_0$$

$$y(x_k) = y_k$$

$$y_{k+1} = y_k + h(a_1 y_{k+1}' + a_0 y_k') + h^2(b_1 y_{k+1}'' + b_0 y_k'')$$

$a_i, b_i = ?$ a.i gr. de volob. se fac cît mai mare

$$x_k = x_0 + k \cdot h, \quad h = \frac{a}{n} \quad [x_0, x_0 + a]$$

$$\left. \begin{array}{l} \text{- from translate, } \Rightarrow x_k = kh \\ \text{- from reduce (h=1)} \end{array} \right\} \Rightarrow x_k = k$$

$$y=1$$

$$1 = \underbrace{1}_{\substack{1 \\ \downarrow}} + h(0+0) + h^2(0+0)$$

$$y=x$$

$$x_{k+1} = x_k + (a_1 + a_0)$$

$$\boxed{a_1 + a_0 = 1}$$

$$y=x^2$$

$$(k+1)^2 = k^2 + (2a_1(k+1) + 2a_0k) + (b_1 \cdot 2 + b_0 \cdot 2)$$

$$k=0 \Rightarrow \boxed{1 = 2a_1 + 2b_1 + 2b_0}$$

$$y=x^3$$

$$(k+1)^3 = k^3 + [3a_1(k+1)^2 + 3a_0k^2] + [6b_1(k+1) + 6b_0k]$$

$$k=0 \Rightarrow \boxed{1 = 3a_1 + 6b_1}$$

$$y=x^4$$

$$(k+1)^4 = k^4 + [4a_1(k+1)^3 + 4a_0k^3] + [12b_1(k+1)^2 + 12b_0k^2]$$

$$k=0 \Rightarrow \boxed{1 = 4a_1 + 12b_1}$$

$$\begin{cases} a_1 + a_0 = 1 \\ a_1 + b_1 + b_0 = \frac{1}{2} \\ a_1 + 2b_1 = \frac{1}{3} \\ a_1 + 3b_1 = \frac{1}{4} \end{cases} \Rightarrow \begin{cases} b_1 = -\frac{1}{12} \\ a_1 = \frac{1}{2} \\ b_0 = \frac{1}{12} \\ a_0 = \frac{1}{2} \end{cases}$$

$$y_{k+1} = y_k + \frac{h}{2} (y'_{k+1} + y'_k) + \frac{h^2}{2} (y''_{k+1} - y''_k)$$

$$y = x^5$$

$$(k+1)^5 = k^5 + \frac{1}{2} (5(k+1)^4 + 5k^4) - \frac{1}{12} (20(k+1)^3 - 20k^3)$$

$$1 = \frac{5}{2} - \frac{20}{12} \cdot 5$$

$$1 \neq \frac{5}{2} - \frac{5}{3}$$

$$y_{k+1} = y_k + h \left(\overbrace{f_{k+1}}^{y_{k+1}} \right) \text{ --- } \\ \text{implicit}$$

$$\left(\overbrace{f_{k,j}}^{j \geq 0} \right) \text{ --- } \\ \text{explicit}$$