

①

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Jacobi $Ax = b$

$$D = \begin{bmatrix} 2 & & \\ & 2 & \\ 0 & & 2 \end{bmatrix}$$

$$E = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = N - P$$

$$N = D, \quad P = E + F$$

$$(N - P)x = b$$

$$Nx = Px + b$$

$$Nx^{(n+1)} = Px^{(n)} + b$$

$$\Rightarrow x^{(n+1)} = \underbrace{N^{-1} \cdot P}_{G} x^{(n)} + N^{-1} \cdot b$$

$$N^{-1} = \begin{bmatrix} \frac{1}{2} & & \\ & \frac{1}{2} & \\ 0 & & \frac{1}{2} \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$$

$$x^{(1)} = G \cdot x^{(0)} + N^{-1} \cdot b = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix}$$

$$x^{(2)} = G \cdot x^{(1)} + N^{-1} \cdot b = \begin{bmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$\|x^{(2)} - x^{(1)}\| = \left\| \begin{bmatrix} 0 \\ \frac{1}{2} \\ 0 \end{bmatrix} \right\| = \frac{1}{2}$$

Spline

$$S_i = a_i + b_i(x-x_i) + c_i(x-x_i)^2 + d_i(x-x_i)^3$$

$$\left\{ \begin{array}{l} S_1(1) = 3 \\ S_1(2) = 5 \\ S_2(2) = 5 \\ S_2(3) = -1 \\ S_1'(2) = S_2'(2) \\ S_1''(2) = S_2''(2) \\ S_1''(1) = 2 \\ S_2''(3) = 3 \end{array} \right\}$$

	1	2	3
f	3	5	-1
f'	2		3

②

$$A = \begin{pmatrix} a & & & \\ b & a & & \\ & b & a & 0 \\ & & \ddots & \ddots \\ 0 & & & b & a \end{pmatrix} \quad d = \begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix}$$

$$-\frac{b}{a}l_1 + l_2$$

$$-\frac{b}{a}l_2 + l_3$$

$$M.x = d$$

for $i = 1:n-1$

$$M(i,:) = M(i+1,:) + M(i,:) * (-M(i+1,i)/a);$$

$$d(i+1) = d(i+1) + d(i) * (-M(i+1,i)/a);$$

$$x(i) = d(i)/a;$$

end

$$x(n) = d(n)/a;$$

for tracing:

$$x(i) = (d(i) - c \cdot x(i+1))/a;$$

Factorizare QR

$$Ax = b$$

$$A = QR$$

$$\downarrow$$
$$QRx = b$$

$$\square \nabla \square = \square$$

$$Q^{-1} = Q^T$$
$$\boxed{Q \cdot Q^T = I_n}$$

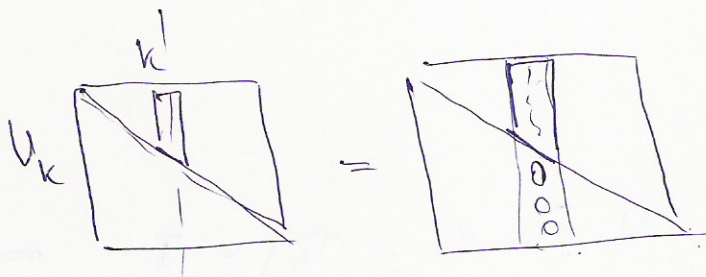
Propri:

- ! 2 matrici ortog $\Rightarrow$ produsul lor e o mat. ortog
o Suma a 2 mat ortog nu e o mat ortog.

$$Rx = Q^T \cdot b$$

1. Reflectorii Householder

$$U_i = I_n - 2 \frac{v \cdot v^T}{\|v\|^2} = I_n - 2 \frac{v \cdot v^T}{v^T \cdot v}$$



$$U_1 = I_m - 2 \cdot \frac{a_1 \cdot a_1^T}{a_1^T \cdot a_1}$$

$$a_1 = A(:, 1)$$

① Se consideră reflectorul Householder

$$U = I - 2uu^T, \quad \|u\| = 1$$

Să se arate că U este ortogonal și un reflector ortog. simetric. Să se det setul de vectori v_i a.t.

$$Uv_i = e_i$$

$$U^T = (I - 2uu^T)^T = I - 2(uu^T)^T = I - 2u \cdot u^T = U \Rightarrow$$

$\Rightarrow U$ - simetric

$$\begin{aligned} U \cdot U^T &= U^2 = (I - 2uu^T)^2 = I - 4uu^T + 4\underbrace{uu^T \cdot uu^T}_{\|u\|=1} = \\ &= I - \cancel{4uu^T} + \cancel{4uu^T} = I \Rightarrow U - \text{ortog} \end{aligned}$$

$$v_i = ?$$

$$U \cdot v_i = e_i$$

$$(I - 2uu^T) \cdot v_i = e_i$$

$$\left. \begin{array}{l} Uv_1 = e_1 \\ Uv_2 = e_2 \\ \vdots \\ Uv_n = e_n \end{array} \right\} \Rightarrow$$

$$U \cdot [v_1 \ v_2 \ \dots \ v_n] = I \Rightarrow$$

$$\Rightarrow [v_1 \ v_2 \ \dots \ v_n] = U \Rightarrow v_i = U(:, i)$$

$$A \in \mathbb{R}^{n \times m}$$

$$n \neq m \quad (n > m)$$

$$A = Q \cdot R$$

$$Q \in \mathbb{R}^{n \times n}$$

$$R \in \mathbb{R}^{n \times m}$$

② Fixe $A \in \mathbb{R}^{n \times m}$ pentru care se cunoaște fact QR,

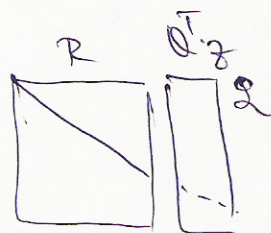
$$A = QR. \quad \bar{A} = [A \mid z] \quad z \in \mathbb{R}^n$$

$$\bar{A} \in \mathbb{R}^{n \times (m+1)}$$

$$\bar{A} \neq \bar{Q} \cdot \bar{R}$$

$$\bar{A} = [QR \mid z]$$

$$Q^T \bar{A} = [R \mid Q^T z] \quad (1)$$



$$\exists U_{n+1} = I_n - 2 \frac{v \cdot v^T}{v^T \cdot v}$$

$$v = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{n+1}$$

$$v = \text{norm}(z_m) \cdot \left(\sqrt{\sum_{i=m+1}^n z_i^2} \right)$$

$$(1) \xrightarrow{U_{n+1}} U_{n+1} Q^T \bar{A} = [R \mid U_{n+1} Q^T z] \Rightarrow \bar{Q}^T \bar{A} = \bar{R}$$

$$\text{Deci } \bar{R} = [R \mid U_{n+1} Q^T z]$$

$$\bar{Q}^T = U_{n+1} Q^T \Rightarrow \bar{Q} = Q \cdot U_{n+1}^T$$

③ Tre matrice

$$P_{ke} = \left[\begin{array}{c|c|c} I_{k-1} & 0 & 0 \\ \hline 0 & c & s \\ \hline 0 & -s & c \\ \hline 0 & 0 & I_{n-k+1} \end{array} \right] \begin{matrix} k \\ e \\ e \\ n-k+1 \end{matrix}, P_{ke} \in \mathbb{R}^{n \times n}$$

$k \quad e$

a. e. $c^2 + s^2 = 1$

a) Arătați că matricea P_{ke} este ortogonală

b) Să se arate că se poate forma următorul sir de matrice

$$P_k = P_{kn} * P_{k,n-1} * \dots * P_{k,k+1}$$

$$P = P_{n+1} \cdot P_{n+2} \cdot \dots \cdot P_1$$

$$PA = R$$

$$P_{ke} * A = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{matrix} k \\ e \\ e \\ n-k+1 \end{matrix}$$

$0 \quad A \in \mathbb{R}^{n \times n}$

$$a_{ij}' = \begin{cases} a_{ij} & , 1 \leq i < k, k < i < l, l < i \leq n \\ c \cdot a_{ik} + s \cdot a_{le} & , i = k \\ -s \cdot a_{ik} + c \cdot a_{le} & , i = l \end{cases}$$

$$a_{ek}' = 0 \Rightarrow \begin{cases} c \cdot a_{le} - s \cdot a_{ek} = 0 \\ c^2 + s^2 = 1 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} c = \frac{a_{ek}}{\sqrt{a_{ek}^2 + a_{le}^2}} \\ s = \frac{a_{le}}{\sqrt{a_{ek}^2 + a_{le}^2}} \\ s = \sqrt{1 - c^2} \end{cases}$$

$$c^2 \cdot a_{le}^2 + s^2 \cdot a_{ek}^2 - 2cs \cdot a_{le} \cdot a_{ek} = 0$$

$$\textcircled{4} \quad A, B \in \mathbb{R}^{n \times n}$$

$$B = A + u v^T \quad u, v \in \mathbb{R}^n$$

$$A = QR$$

$$B \stackrel{?}{=} Q R_B$$

$$B = QR + u v^T$$

$$B = QR + Q Q^T \cdot u v^T$$

$$B = Q (R + Q^T u v^T)$$

$$\exists P \text{ a.s.} \quad P(R + Q^T u v^T) = \nabla$$

$$\Rightarrow \begin{cases} Q_B = Q \cdot P^T \\ R_B = P(R + Q^T u v^T) \end{cases}$$