

Inversarea sistemelor liniare

$$A \cdot x = b \quad A \in \mathbb{R}^{n \times n}, \quad b, x \in \mathbb{R}^n$$

$$x = A^{-1} \cdot b$$

$$\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \begin{bmatrix} \square \\ \square \end{bmatrix} \longrightarrow \begin{bmatrix} \triangle & \square \\ \square & \square \end{bmatrix} = \begin{bmatrix} \square \\ \square \end{bmatrix}$$

$$x_i = \frac{b_i - \sum_{j=i+1}^n a_{ij} \cdot x_j}{a_{ii}}$$

① $A \in \mathbb{R}^{n \times n}$ inv. mat. prin met iter.

$$A_k = \begin{bmatrix} A_{k-1} & u \\ v^T & a_{kk} \end{bmatrix}$$

$$A_{k-1} \in \mathbb{R}^{(k-1) \times (k-1)}$$

$$u, v \in \mathbb{R}^{k-1}$$

$$a_{kk} \in \mathbb{R} \longrightarrow A_k^{-1}$$

$$k=n \Rightarrow A^{-1}$$

$$\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} = \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix}$$

$$B_k \in \mathbb{R}^{k \times k} \text{ a.t. } A_k \cdot B_k = I_k$$

$$\begin{bmatrix} A_{k-1} & u \\ v^T & a_{kk} \end{bmatrix} \cdot B_k = I_k$$

$$B_k = \begin{bmatrix} B_{k-1} & x \\ y^T & z \end{bmatrix} \begin{matrix} k-1 \\ 1 \end{matrix}$$

$$\begin{bmatrix} A_{k-1} & u \\ v^T & a_{kk} \end{bmatrix} \cdot \begin{bmatrix} B_{k-1} & x \\ y^T & z \end{bmatrix} = \begin{bmatrix} I_{k-1} & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A_{k-1} \cdot B_{k-1} + u u^T & A_{k-1} \cdot x + u z \\ v^T B_{k-1} + a_{kk} y^T & v^T x + a_{kk} z \end{bmatrix} = \begin{bmatrix} \dot{I}_{k-1} & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{cases} A_{k-1} B_{k-1} + u u^T = \dot{I}_{k-1} \\ A_{k-1} \cdot x + u z = 0 \\ v^T B_{k-1} + a_{kk} y^T = 0 \\ v^T x + a_{kk} z = 1 \end{cases}$$

$$A_{k-1}^{-1} \cdot A_{k-1} \cdot B_{k-1} + A_{k-1}^{-1} \cdot u \cdot y^T = A_{k-1}^{-1}$$

$$\boxed{B_{k-1} = A_{k-1}^{-1} - A_{k-1}^{-1} u y^T} \quad (2)$$

$$v^T A_{k-1}^{-1} - \underbrace{v^T A_{k-1}^{-1} u}_{\text{scalar}} y^T + a_{kk} y^T = 0$$

$$\boxed{y^T = \frac{v^T A_{k-1}^{-1}}{v^T A_{k-1}^{-1} u - a_{kk}}} \quad (1)$$

$$x = -A_{k-1}^{-1} u z \quad (2)$$

$$-v^T A_{k-1}^{-1} u z + a_{kk} z = 1 \Rightarrow z = \frac{1}{a_{kk} - v^T A_{k-1}^{-1} u}, \quad (1)$$

$$A_k = \begin{bmatrix} I_{k-1} & u \\ u^T & 1 \end{bmatrix}$$

$$\downarrow$$

$$1 - u^T u \neq 0$$

$$\|u\|_2 \neq 1$$

②

$$(A + u v^T)^{-1}$$

$$A, A^{-1} \in \mathbb{R}^{n \times n}$$

$$u, v \in \mathbb{R}^n$$

$$A \in \mathbb{R}^{n \times n} \text{ invertierbar}$$

$$A^{-1}$$

$$u, v, x, y, b \in \mathbb{R}^n$$

$$A \cdot x = b$$

$$(A + u v^T) \cdot y = b \quad \Bigg| \Rightarrow \quad A y + \underbrace{u v^T \cdot y}_k = A \cdot x$$

$$A^{-1} / \quad A(y - x) = -k \cdot u$$

$$y = x - k \cdot A^{-1} \cdot u$$

$$(A + u v^T) \cdot (x - k \cdot A^{-1} \cdot u) = b$$

$$\cancel{A \cdot x} - k \cdot \cancel{A A^{-1} \cdot u} + \underbrace{u v^T \cdot x} - k \cdot \underbrace{u v^T \cdot A^{-1} \cdot u} = \cancel{b}$$

$$-k \cdot \underbrace{u} + \underbrace{v^T \cdot x \cdot u} - k \cdot \underbrace{v^T \cdot A^{-1} \cdot u \cdot u} = 0$$

$$(-k + v^T \cdot x - k \cdot v^T \cdot A^{-1} \cdot u) \cdot u = 0$$

$$k = \frac{v^T \cdot x}{1 + v^T \cdot A^{-1} \cdot u}$$

$$y = x - A^{-1} \cdot u \cdot \frac{v^T \cdot x}{1 + v^T \cdot A^{-1} \cdot u}$$

$$(A + u v^T)^{-1} \cdot b = A^{-1} \cdot b - A^{-1} \cdot u \cdot \frac{v^T \cdot A^{-1} \cdot b}{1 + v^T \cdot A^{-1} \cdot u} \Rightarrow$$

$$\boxed{(A + u v^T)^{-1} = A^{-1} - A^{-1} \cdot u \cdot \frac{v^T \cdot A^{-1}}{1 + v^T \cdot A^{-1} \cdot u}}$$

$$A \cdot b = B \cdot b \Rightarrow \text{0 possible nur } A = B$$

$$\dot{I} + u v^T$$

$$\dot{I} + u u^T$$

$$\dot{I} + \alpha \cdot u v^T$$

$$|u \cdot u^T| = 0$$

③

$$A \in \mathbb{R}^{n \times n}$$

$$A \cdot x = b$$

$$(u_i)_{i=1:n}$$

vektori A-ortogonalni

$$\langle A u_i, u_j \rangle = 0, \quad i \neq j$$

$$\langle A u_i, u_i \rangle > 0$$

$$x = \sum_{i=1}^n c_i u_i, \quad c_i = ? \quad i=1:n$$

$$A \cdot \sum_{i=1}^n \lambda_i u_i = 0 \Leftrightarrow \lambda_i = 0, \quad i=1:n$$

$$\Rightarrow \sum_{i=1}^n A \lambda_i u_i = 0$$

$$\langle \sum_{i=1}^n A \lambda_i u_i, u_j \rangle = 0 = \sum_{i=1}^n \lambda_i \langle A u_i, u_j \rangle =$$

$$= \lambda_j \underbrace{\langle A u_j, u_j \rangle}_{>0} \Rightarrow$$

$$\Rightarrow \lambda_j = 0, \quad j=1:n$$

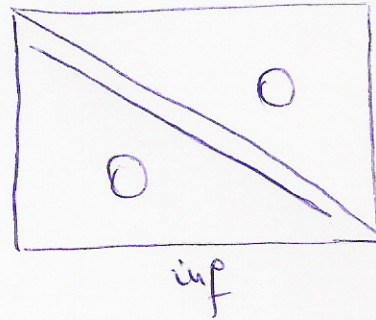
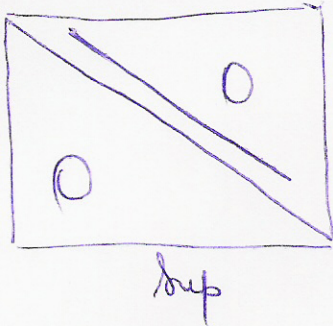
$$A \cdot \sum_{i=1}^n c_i u_i = b \Rightarrow \sum_{i=1}^n A c_i u_i = b$$

$$\langle b, u_j \rangle = \langle \sum_{i=1}^n A c_i u_i, u_j \rangle = \sum_{i=1}^n \langle A c_i u_i, u_j \rangle =$$

$$= \sum_{i=1}^n c_i \langle A u_i, u_j \rangle = c_j \langle A u_j, u_j \rangle$$

$$\Rightarrow c_j = \frac{\langle b, u_j \rangle}{\langle A u_j, u_j \rangle}$$

④ So se adapteze Met. Gauss ptr. o matrice
superior Hessenberg



$$\begin{cases} x_n = \frac{b_n}{a_{nn}} \\ x_i = \frac{b_i - \sum_{j=i+1}^n a_{ij} x_j}{a_{ii}} \end{cases}$$

$$i = 1:n-1$$

$$\begin{cases} x_n = \frac{b_n}{a_{nn}} \\ x_i = \frac{b_i - a_{i,i+1} x_{i+1}}{a_{ii}} \end{cases}$$

$$i = 1:n-1$$

function $x = \text{hsolve}(a, b)$

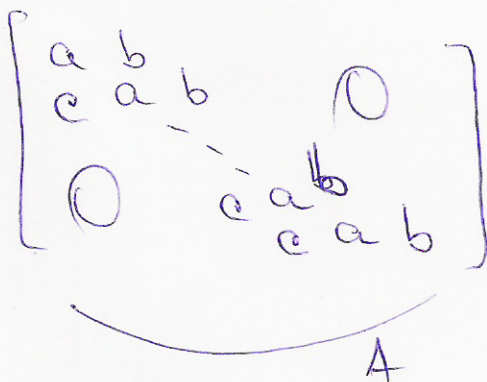
$n = \text{length}(b)$

$x(n) = b(n) / a(n, n);$

for $i = n-1 : -1 : 1$

$x(i) = (b(i) - a(i, i+1) * x(i+1)) / a(i, i);$

end



$$(a, b, c, v)$$

$$A \cdot x = v$$

$$x_n, x_{n-1}$$