

5p prezentare } 1p.
5p test

1 temă de casă (5 săptămâni) 2p

Norme vectoriale și matriciale

$$\|\cdot\|: \mathbb{R}^n \longrightarrow \mathbb{R}_+$$

$$\forall x, y \in \mathbb{R}^n, \alpha \in \mathbb{R}$$

$$1) \|x\| \geq 0 \text{ și } \|x\| = 0 \Leftrightarrow x = 0_{\mathbb{R}^n}$$

$$2) \|\alpha x\| = |\alpha| \cdot \|x\|$$

$$3) \|x+y\| \leq \|x\| + \|y\|$$

Formula Holder

$$x \in \mathbb{R}^n$$

$$\|x\|_p = \left(\sum_{i=1}^n |x_i|^p \right)^{1/p}$$

$$\|x\|_1 = \sum_{i=1}^n |x_i|$$

$$\|x\|_2 = \sqrt{\langle x, x \rangle}$$

$$\|x\|_\infty = \lim_{p \rightarrow \infty} \left(\sum_{i=1}^n |x_i|^p \right)^{1/p} = \lim_{p \rightarrow \infty} \left(\max_{i=1, \dots, n} \left(\frac{|x_i|}{M} \right)^p \right)^{1/p} =$$

fie $M = \max_i |x_i|$

$$= M = \max_i |x_i|$$

Norme matricielle

$$A \in \mathbb{R}^{l \times l}$$

a) Frobenius

$$\|A\|_F = \sqrt{\sum_i \sum_j A_{ij}^2}$$

b) Norme subordonnée

$$\|A\|_p = \sup_{x \neq 0} \frac{\|Ax\|_p}{\|x\|_p}$$

$$\|A\|_1 = \max_i \sum_{j=1}^n |A_{ij}|$$

$$\|A\|_\infty = \max_j \sum_{i=1}^n |A_{ij}|$$

$$\|A\|_2 = \sqrt{\rho(A^T A)}$$

nr de conditionnement

$$\kappa_p(A) = \|A\|_p \|A^{-1}\|_p$$

$$p \in \{F, \mathbb{N}\}$$

$$Ax = b$$

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$$

$$A \rightarrow A + \delta A$$

$$\delta A = \begin{bmatrix} 0,001 & -0,027 \\ 0,034 & -0,019 \end{bmatrix}$$

$$x \rightarrow x + \delta x$$

$$(A + \delta A)(x + \delta x) = b$$

$$\cancel{Ax} + A\delta x + \delta Ax + \delta A\delta x = \cancel{b}$$

$$A\delta x + \delta Ax + \delta A\delta x = 0$$

$$\boxed{\Sigma_A = \frac{\|\delta A\|}{\|A\|}}$$

eroare rel.

$$\textcircled{1} \quad \delta A \delta x \approx 0$$

$$\Rightarrow A\delta x + \delta Ax = 0$$

$$\Leftrightarrow A^{-1} / A\delta x = -\delta Ax$$

$$\delta x = -A^{-1} \delta Ax$$

$$\|\delta x\| = \|A^{-1} \delta Ax\| \leq \|A^{-1}\| \cdot \|\delta A\| \cdot \|x\|$$

$$\Rightarrow \frac{\|\delta x\|}{\|x\|} \leq \|A^{-1}\| \cdot \frac{\|\delta A\|}{\|A\|} = \|A^{-1}\| \cdot \|A\| \cdot \frac{\|\delta A\|}{\|A\|}$$

$$\Sigma_x \leq K(A) \cdot \Sigma_A$$

Dacă $K(A)$ e de ord. unității $\Rightarrow$ precizie ader.

$$\textcircled{2} \quad \delta A \delta x \neq 0$$

$$A \delta x + \delta A x + \delta A \delta x = 0$$

$$A^{-1} / A \delta x = -\delta A x - \delta A \delta x$$

$$\delta x = -A^{-1} (\delta A x + \delta A \delta x)$$

$$\|\delta x\| \leq \|A^{-1}\| (\|\delta A\| \|x\| + \|\delta A\| \|\delta x\|)$$

$$\|\delta x\| (1 - \|A^{-1}\| \|\delta A\|) \leq \|A^{-1}\| \|\delta A\| \|x\|$$

$$\varepsilon_x \leq \frac{\kappa(A) \cdot \varepsilon_A}{1 - \kappa(A) \cdot \varepsilon_A}$$

Introducere în MATLAB

>> 1

ans =

1

>> help site

>> a=1.

a =
1

>> 7+3

ans =

10

>> save figier.

>> load figier

>> whos

1 [1]
'ano' [a n a]

Lucrul cu vectori

$$\Rightarrow x = [2 \ 7 \ -1 \ 3 \ 0]$$

$$y = [\quad \quad \quad]$$

→ vect. coloană

$$x = [2; 7; -1; 3; 0]$$

Sau

$$x = x' \text{ (transpus)}$$

$x(i)$

; la sfârșitului ambele
apărute

$$\gg x+1$$

$$\gg \text{size}(x)$$

$$\gg \text{sum}(x)$$

$$\gg x < 2$$

$$\gg \text{sum}(x < 2)$$

$$\gg x+y$$

$$\gg x = 1:10$$

$$\gg x = 1:2:11$$

$$\gg x = 1:0.1:10$$

pi

eps

inf

NaN

i, j

$$1 + \text{eps} \neq 1$$

$$1/0, \neq 0$$

$$0/0, \infty/0$$

Matrice

$$\Rightarrow A = \begin{bmatrix} 2 & 3 & 7 \\ 0 & 0 & 1 \\ 2 & 3 & -2 \end{bmatrix};$$

$$A+B$$

$$A-B$$

$$A \times B$$

Hodoword

$$A \circ B \quad (a_{ii} \times b_{ii}, \dots)$$

$$A \cdot B \quad (\text{analog})$$

$$A \cdot -B \quad (\text{analog})$$

$$A \cdot ^B \quad (a_{ii}^{b_{ii}}, \dots)$$

$$A \cdot ^2$$

$$A \times x'$$

$$x \times x'$$

$$\text{norm}(x, p) \rightarrow 2$$

$$\text{norm}(A, p)$$

$$\text{disp}(x) \quad (\text{în ocupă sf. de memorie})$$

$$A = \text{zero}(n, m)$$

$$(n)$$

$$A = \text{eye}(n)$$

mat. unitate

$$A(i, j)$$

$$i = [1 \ 7 \ 3 \ 9]$$

$$j = [1 \ 2 \ 2 \ 3]$$

$$A(i, j)$$

$$i = 2:7$$

$$j = 3:9$$

$$A(i, j)$$



$A(:, 7) \rightarrow$ scoate col 7

$A(3) \Leftrightarrow A(3, :)$

$A(:, :)$ $\rightarrow$ toată mat.

$A(:)$ $\rightarrow$ toate elementele pe o linie

$A < 4$

for w
┌
├
├
└

... cont pe rândul urm

>> for

if expr
instr
else
instr
end

if expr
instr
elseif expr
instr
end

ex: if $(x > 2) \& (x \leq 10)$
disp(—)
else
disp(—)
end

for $x = \langle \text{vector} \rangle$

instr.

end.

ex:

for $i = 1:n$

$i = n:-1:1$

end.

$$\epsilon_x \leq \kappa(A) \cdot \frac{\|\delta A\|}{\|A\|}$$

$$A = \begin{bmatrix} 2 & 3 & ; & 1 & 2 \end{bmatrix}$$

$$\delta A = \begin{bmatrix} 0,01 & -0,02 & ; & 0,09 & -0,03 \end{bmatrix}$$

$$\epsilon_x = \text{Cond}(A) \times \text{Norm}(\delta A) / \text{Norm}(A) ;$$

Nice pr.m