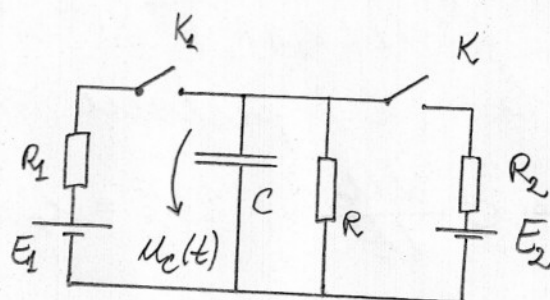


24. Mar. 2008

Tema 1 ED

Problema P1:

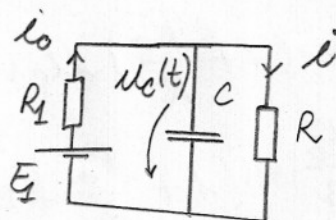
$E_1 = 10V$; $E_2 = 20V$, $t_1 = 100\mu s$,
 $t_2 = 200\mu s$; $t_3 = 300\mu s$
 $R_1 = 6k\Omega$; $R_2 = 4k\Omega$; $R = 4k\Omega$,
 $C = 80nF$.



Rezolvare:

i) $t = 0 \Rightarrow K_1 \downarrow$ (închis) și C se
va încărca prin R și R_2 .

În operational avem:



$u_C(s) = R \cdot i(s)$, dar $i_0(s) = i(s) (1 + sCR)$,

iar $E_1(s) - R_1 i_0(s) = u_C(s) = R i(s)$

$$\Rightarrow \frac{E_1}{s} = R \dot{i}(s) + R_1 i(s) (1 + sCR) = i(s) (R + R_1 + sCRR_1)$$

$$\Rightarrow \dot{i}(s) = \frac{E_1}{CRR_1 s (s + \frac{R+R_1}{CRR_1})} = \frac{E_1}{CRR_1} \cdot \frac{CRR_1}{R+R_1} \left(\frac{1}{s} - \frac{1}{s + \frac{R+R_1}{CRR_1}} \right)$$

$$\Rightarrow i(s) = \frac{E_1}{R+R_1} \left(\frac{1}{s} - \frac{1}{s + \frac{R+R_1}{CRR_1}} \right). \text{ Obținem:}$$

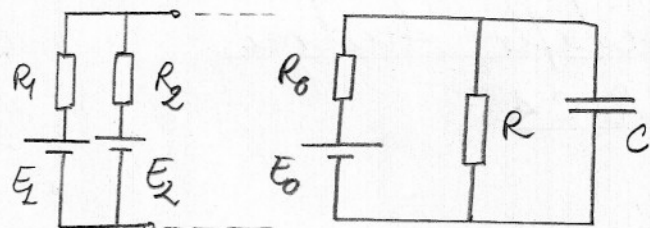
$$\begin{cases} i(t) = \frac{E_1}{R+R_1} (1 - e^{-t/\tau}), \text{ cu } \tau = \frac{CRR_1}{R+R_1} = C(R \parallel R_1) \\ u_C(t) = \frac{E_1 R}{R+R_1} (1 - e^{-t/\tau}), \text{ cu } t \in [0, t_1] \end{cases}$$

ii) $t = t_1 \Rightarrow K_2 \downarrow$ (inchis)

Putem face o echivalare

Thevenin:

$(E_1, R_1), (E_2, R_2) \rightarrow (E_0, R_0)$



cu $E_0 = \frac{E_2/R_2 + E_1/R_1}{1/(R_1 \parallel R_2)}$ și $R_0 = R_1 \parallel R_2$. Cu montajul de la (i) putem scrie:

$$i_c = C \frac{dU_c}{dt}; U_c = i_R \cdot R \text{ și } i_0 = i_c + i_R$$

$$\text{Dar } E_0 - R_0 i_0 = U_c \Rightarrow E_0 = U_c + R_0 C \frac{dU_c}{dt} + R_0 \frac{U_c}{R}$$

$$\Rightarrow E_0 = U_c (1 + R_0/R) + R_0 C \frac{dU_c}{dt}. \text{ În operational:}$$

$$\frac{E_0}{s} = (1 + \frac{R_0}{R}) U_c(s) + R_0 C (s U_c(s) - U_{01})$$

$$\Rightarrow \frac{E_0}{s} = (1 + \frac{R_0}{R} + R_0 C s) U_c(s) - R_0 C U_{01}$$

$$\Rightarrow U_c(s) = \frac{E_0}{s(1 + \frac{R_0}{R} + R_0 C s)} + \frac{R_0 C U_{01}}{1 + \frac{R_0}{R} + R_0 C s}$$

$$\Rightarrow U_c(s) = \frac{E_0 R}{R + R_0} \left(\frac{1}{s} - \frac{1}{s + \frac{R + R_0}{R R_0 C}} \right) + \frac{U_{01}}{s + \frac{R + R_0}{R R_0 C}}$$

$$\Rightarrow U_c(t) = \frac{E_0 R}{R + R_0} + \left(U_{01} - \frac{E_0 R}{R + R_0} \right) e^{-t^*/\tau_2}$$

$$\text{cu } \tau_2 = C(R_0 \parallel R), \text{ iar } t \in (t_1, t_2]$$

$$i(t) = i_R(t) = \frac{U_c(t)}{R} \Rightarrow i_R(t) = \frac{E_0}{R + R_0} + \left(\frac{U_{01}}{R} - \frac{E_0}{R + R_0} \right) e^{-t^*/\tau_2}$$

$$U_{01} = \lim_{t \uparrow t_1} U_c(t) = \frac{E_1 R}{R + R_1} (1 - e^{-t_1/\tau_1}),$$

$$\text{iar } t^* = t - t_1$$

iii) $t = t_2 \Rightarrow K_1 \uparrow$ (deschis) și prin analogie cu ii) avem:

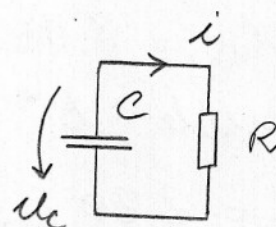
$$\begin{cases} u_C(t) = \frac{E_2 R}{R+R_2} + (u_{02} - \frac{E_2 R}{R+R_2}) e^{-t^*/\tau_3} \\ i_R(t) = \frac{E_2}{R+R_2} + (u_{02}/R - \frac{E_2}{R+R_2}) e^{-t^*/\tau_3} = i(t), \end{cases}$$

unde $t \in [t_2, t_3)$, $\tau_3 = C(R \parallel R_2)$, iar: $t^* = t - t_2$,

$$u_{02} = \lim_{t \nearrow t_2} u_C(t) = \frac{E_2 R}{R+R_0} + (u_{01} - \frac{E_2 R}{R+R_0}) e^{-t_2/\tau_2}$$

iv) $t = t_3 \Rightarrow K_2 \uparrow$ (deschis)

$$u_C(t) = +i(t) \cdot R, \text{ cu } -i(t) = C \frac{du_C(t)}{dt}$$



$$\Rightarrow u_C(t) = -CR \frac{du_C(t)}{dt}$$

$$\Rightarrow u_C(s) = -CR(su_C(s) - u_{03})$$

$$\Rightarrow u_C(s) = \frac{u_{03}}{s+1/(CR)} \Rightarrow u_C(t) = u_{03} e^{-t^*/\tau_4},$$

cu $t \in [t_3, \infty)$, $\tau_4 = CR$ și:

$$u_{03} = \lim_{t \nearrow t_3} u_C(t) = \frac{E_2 R}{R+R_2} + (u_{02} - \frac{E_2 R}{R+R_2}) e^{-t_3/\tau_3}$$

$$i(t) = \frac{u_C(t)}{R} = \frac{u_{03}}{R} e^{-t^*/\tau_4}, \text{ unde } t^* = t - t_3$$

Acum putem înlocui numeric:

i) $\tau_4 = 192 \mu s$ și:

$$u_C(t) = 4(1 - e^{-t/\tau_4}) \text{ [V]}$$

$$i(t) = 1 - e^{-t/\tau_4} \text{ [mA]}$$

$$u_{01} \approx 1.62 \text{ V}, i_{01} \approx 0.4 \text{ mA},$$

unde $t \in [0, t_1]$

ii) $E_0 = 16 \text{ V}$, $R_0 = 2.4 \text{ k}\Omega$, $\tau_2 = 120 \mu\text{s}$ și:

$$\begin{cases} u_C(t) = 10 - 8.38 e^{-t^*/\tau_2} \text{ [V]} \\ i(t) = 2.5 - 2.1 e^{-t^*/\tau_2} \text{ [mA]} \\ u_{02} \approx 6.36 \text{ V}, i_{02} \approx 1.59 \text{ mA} \\ \text{unde } t^* = t - t_1 \end{cases}$$

iii) $\tau_3 = 160 \mu\text{s}$ și:

$$\begin{cases} u_C(t) = 10 - 3.64 e^{-t^*/\tau_3} \text{ [V]} \\ i(t) = 2.5 - 0.91 e^{-t^*/\tau_3} \text{ [mA]} \\ u_{03} \approx 8.05 \text{ V}; i_{03} \approx 2.01 \text{ mA} \\ \text{unde } t^* = t - t_2 \end{cases}$$

iv) $\tau_4 = 320 \mu\text{s}$, $t^* = t - t_3$

$$\begin{cases} u_C(t) = 8.05 e^{-t^*/\tau_4} \text{ [V]} \\ i(t) \approx 2 e^{-t^*/\tau_4} \text{ [mA]} \end{cases}$$

Grafic, avem:

