Quantum Computation & Cryptography
Day 5

Quantum hardware

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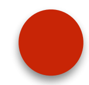
Recall

States	Representing the state of the system	$\mathcal{S} \quad s \in \mathcal{S}$
Transformations	Changing states in time	$\mathcal{S} \to \mathcal{S}$
Composition	The state of multiple systems	$\mathcal{S}_{AB} = \mathcal{S}_A \otimes \mathcal{S}_B$
Observation (measurement)	Observing physical properties	$\mathcal{S} \to \mathbb{R}$ $\mathcal{S} \to [0,1]$

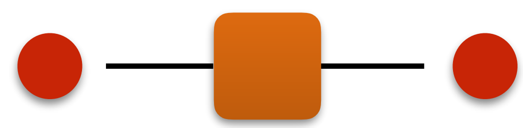
Recap

States



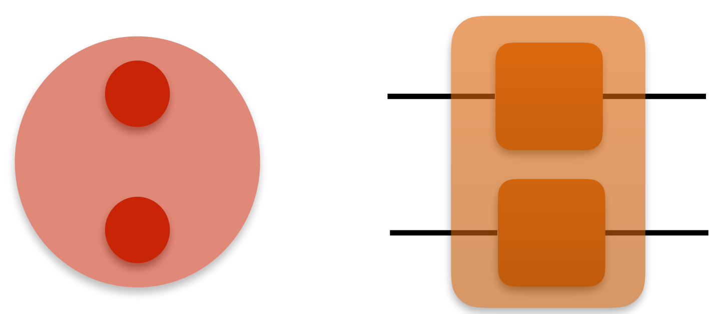
$\mathcal{S} \quad s \in \mathcal{S}$

Transformations



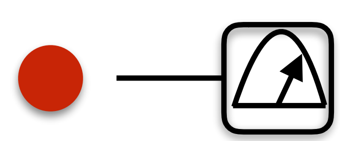
$\mathcal{S} \to \mathcal{S}$

Composition



$\mathcal{S}_{AB} = \mathcal{S}_A \otimes \mathcal{S}_B$

Observation (measurement)



$\mathcal{S} \to \mathbb{R}$

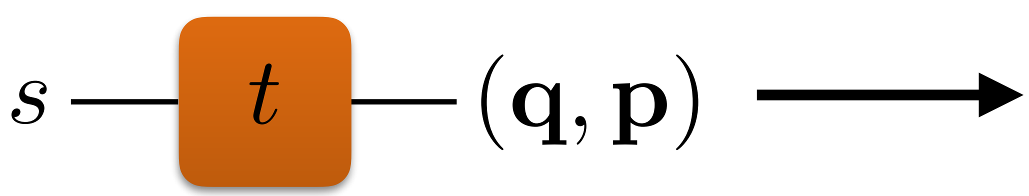
$\mathcal{S} \to [0, 1]$

Recap - classical mechanics

States $\mathcal{S} = \mathbb{R}^{dN}$ $s = (\mathbf{q}_1, \mathbf{q}_2, ..., \mathbf{q}_N, \mathbf{p}_1, \mathbf{p}_2, ...\mathbf{p}_N)$

$\mathbf{q}_i = (q_i^1, q_i^2, ...q_i^d)$ $\mathbf{p}_i = (p_i^1, p_i^2, ...p_i^d)$

Transformations $H(\mathbf{q}_1, ...\mathbf{q}_N, \mathbf{p}_1, ...\mathbf{p}_N, t)$



$$\frac{d\mathbf{p}}{dt} = -\frac{\partial H}{\partial \mathbf{q}} \qquad \frac{d\mathbf{q}}{dt} = \frac{\partial H}{\partial \mathbf{p}}$$

Composition $\mathcal{S}_{AB} = \mathcal{S}_A \times \mathcal{S}_B$ $s_{AB} = s_A \cdot s_B$

Observation (measurement)

Pretty much any “well-behaved” function of the form:

$$f : \mathcal{S} \to \mathbb{R}$$

Recap - quantum mechanics

States	Unit vectors in a complex vector space	$\vert\psi\rangle \in \mathcal{H}$ $\vert\vert\psi\rangle\vert^2 = 1$
Transformations	Schroedinger's equation	$H\vert\psi\rangle = i\hbar\frac{d\vert\psi\rangle}{dt}$
Composition	Tensor product	$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$
Observation (measurement)	Hermitian operators	$O = O^\dagger$

From classical to quantum

There is a way to take any classical system and quantise it

Canonical quantisation

Take a classical Hamiltonian

$$H = f(\mathbf{q}_1, ...\mathbf{q}_N, \mathbf{p}_1, ...\mathbf{p}_N, t)$$

Turn all p's and q's into Hermitian operators and impose canonical commutation relations

$\mathbf{q}_i \to \hat{\mathbf{q}}_i \quad \mathbf{p}_i \to \hat{\mathbf{p}}_i$ such that $$[\hat{\mathbf{q}}_i, \hat{\mathbf{p}}_i] = i\hbar$$ $$[\hat{\mathbf{q}}_i, \hat{\mathbf{p}}_j] = 0, i \neq j$$

The quantum Hamiltonian is then just

$$\hat{H} = f(\hat{\mathbf{q}}_1, ...\hat{\mathbf{q}}_N, \hat{\mathbf{p}}_1, ...\hat{\mathbf{p}}_N, t)$$

From quantum to classical

How do we go back to classical?

Take expectation values of the quantum operators

$$\mathbf{q}_i \leftarrow \langle \hat{\mathbf{q}}_i \rangle \qquad \mathbf{p}_i \leftarrow \langle \hat{\mathbf{p}}_i \rangle$$

Where $\langle O \rangle = \langle \psi | O | \psi \rangle$

Ehrenfest's theorem

Why does this work?

From quantum to classical

Recall…

$$\left(\sum_i a_i\right)^2 = \sum_i a_i^2 + \boxed{\sum_{i \neq j} a_i a_j}$$

interference term

Interference term close to 0 → classical computation

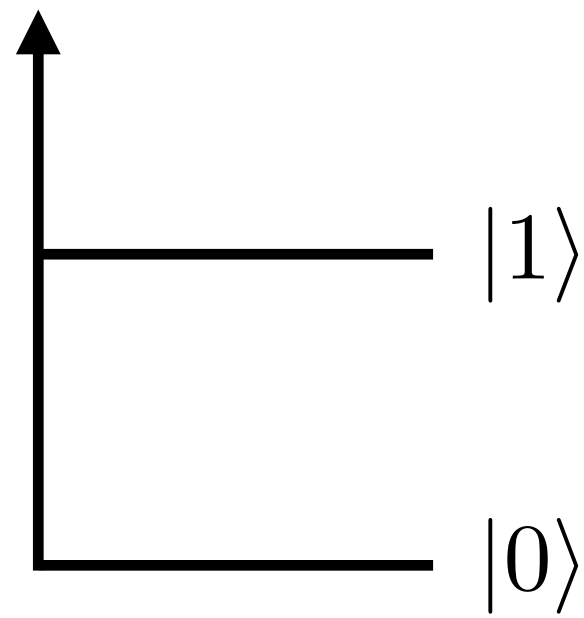
For everyday phenomena, amplitudes are fairly random

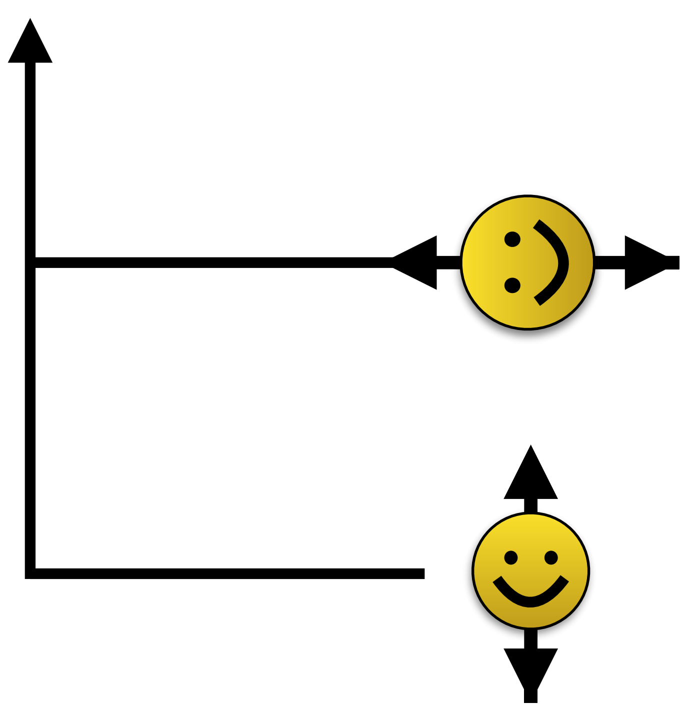
Interference term is close to 0

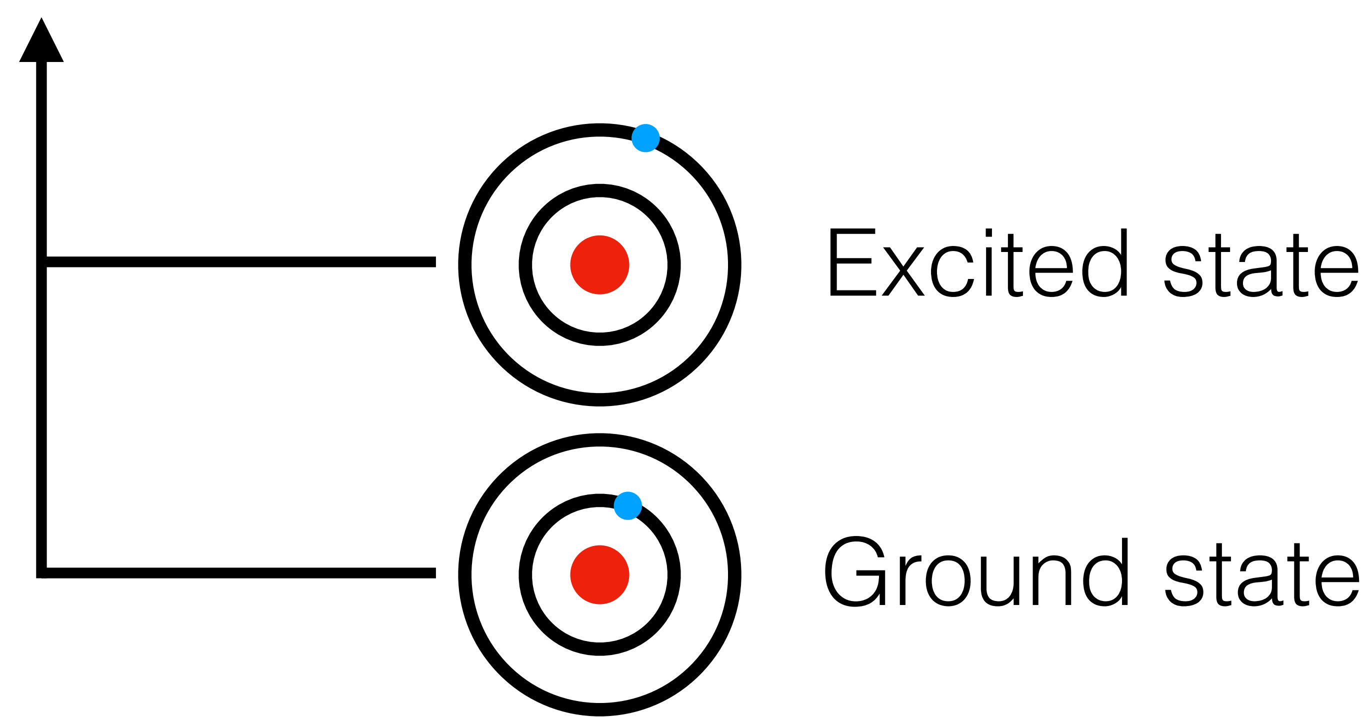
This is why we don't see quantum weirdness around us!

The qubit (again)

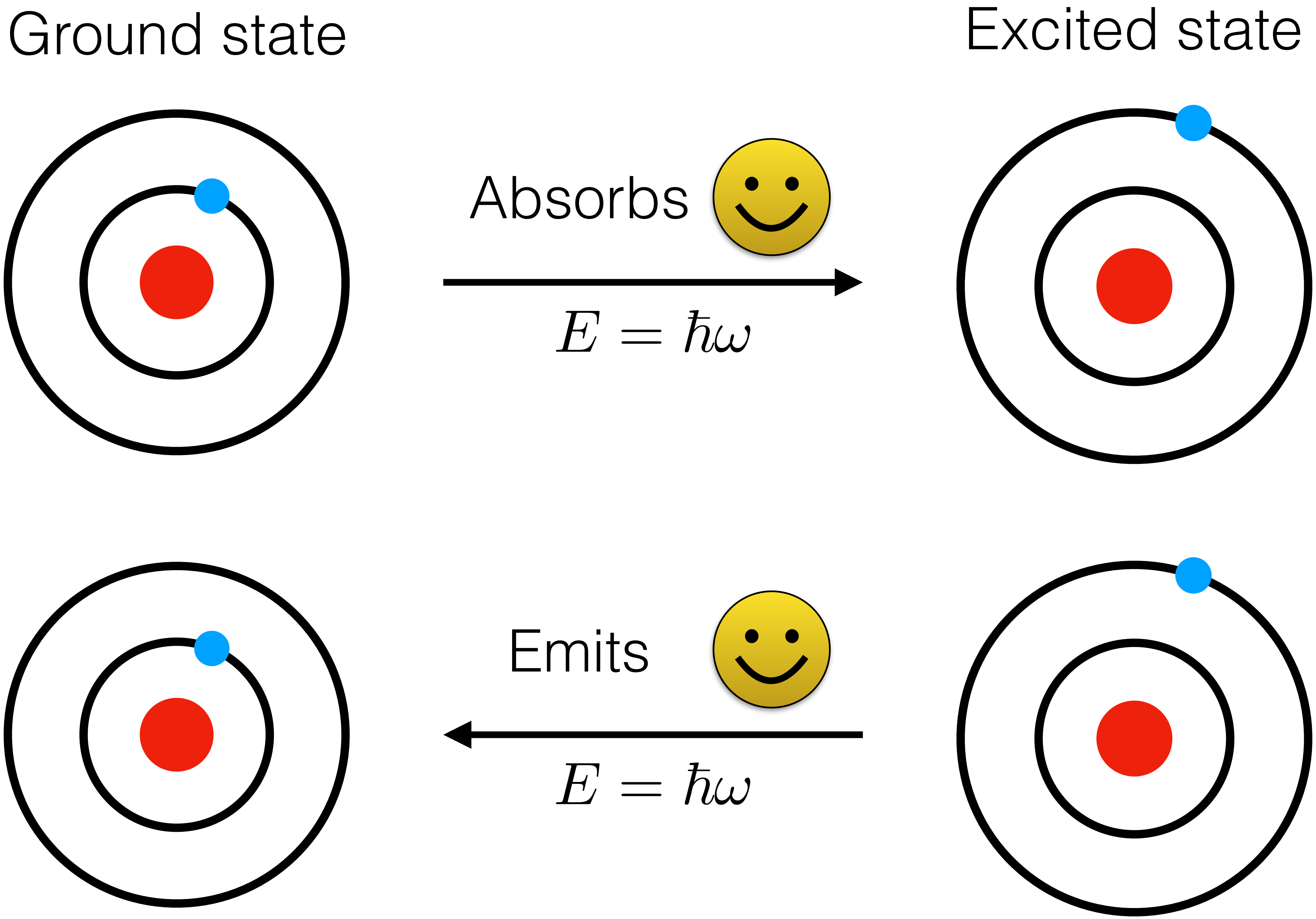
Essentially a 2-level quantum system





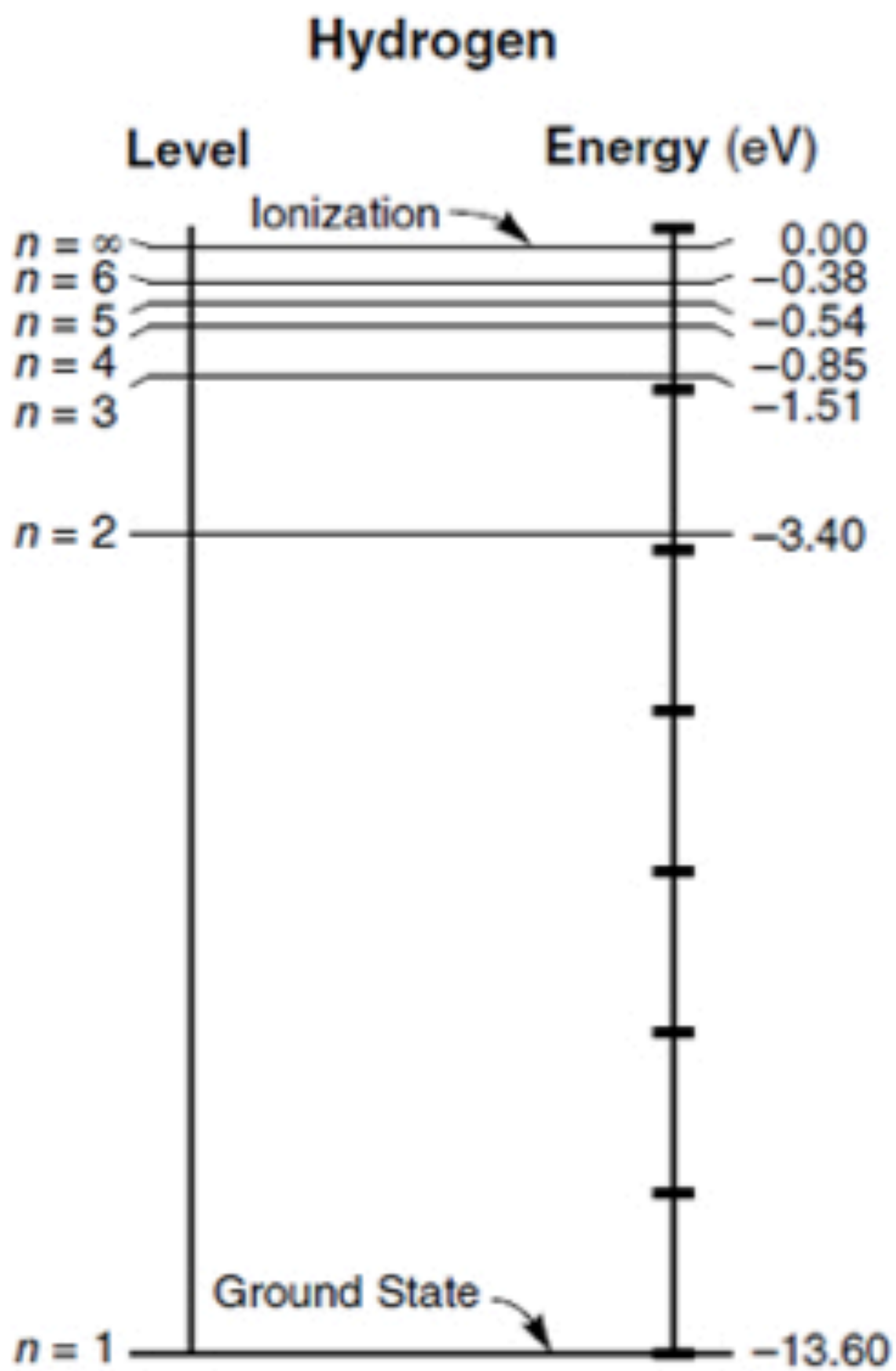


Atoms

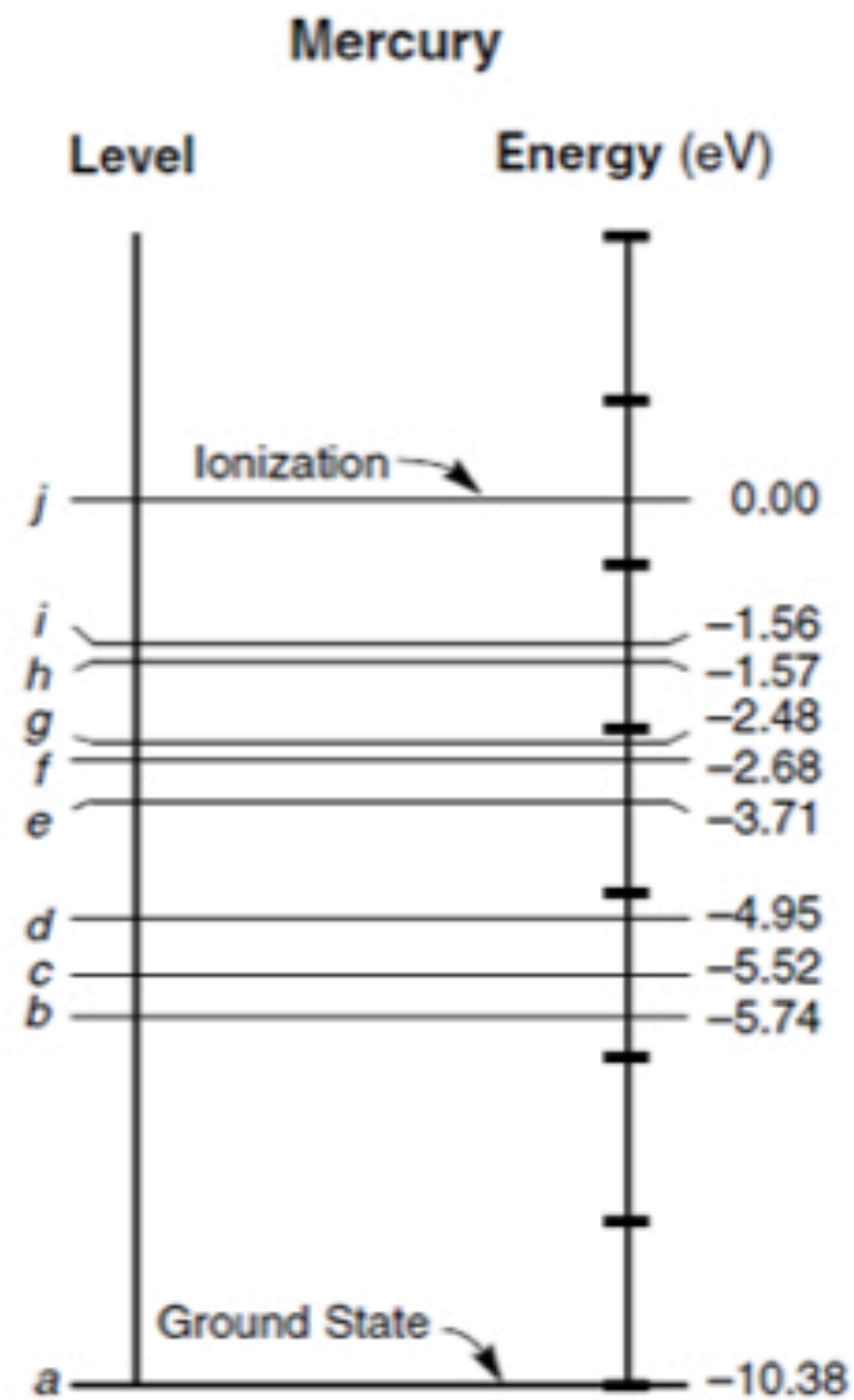


Ground state
Excited state
Absorbs
$E = \hbar\omega$
Emits
$E = \hbar\omega$

Atoms



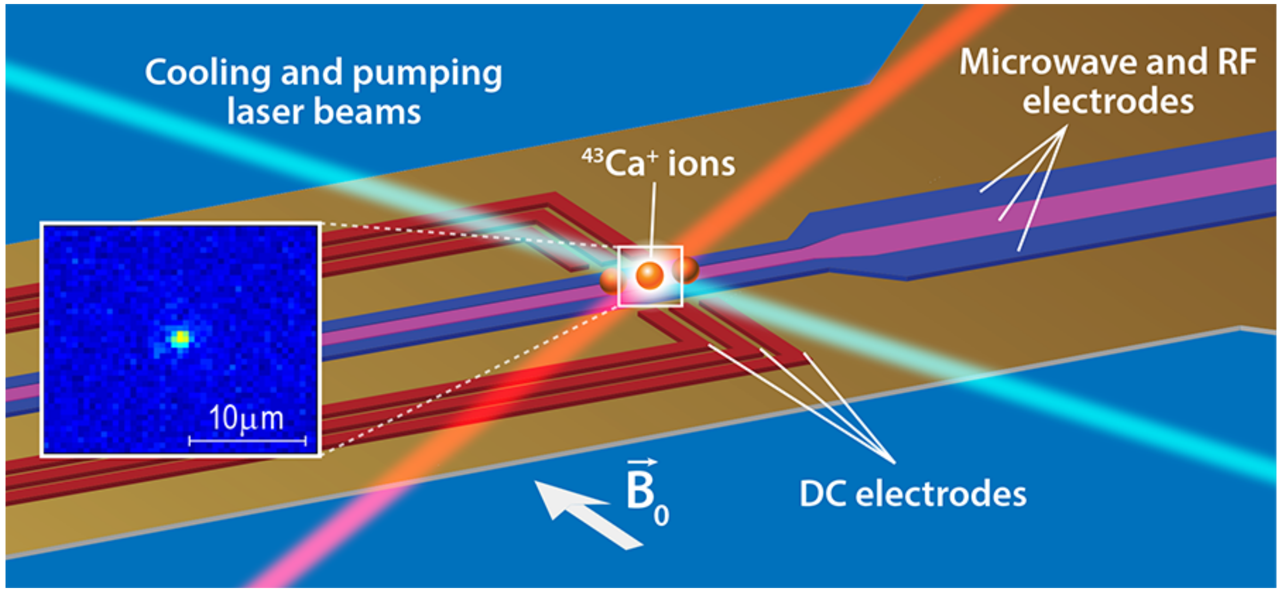
Energy Levels for the Hydrogen Atom



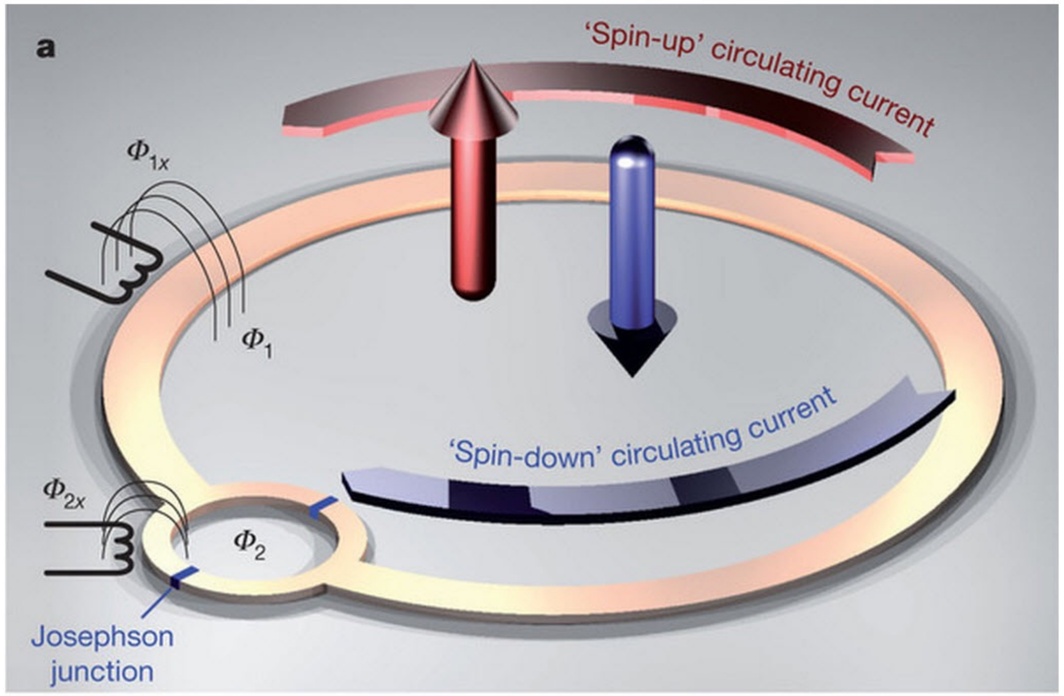
A Few Energy Levels for the Mercury Atom

Qubits as atoms

Natural atoms

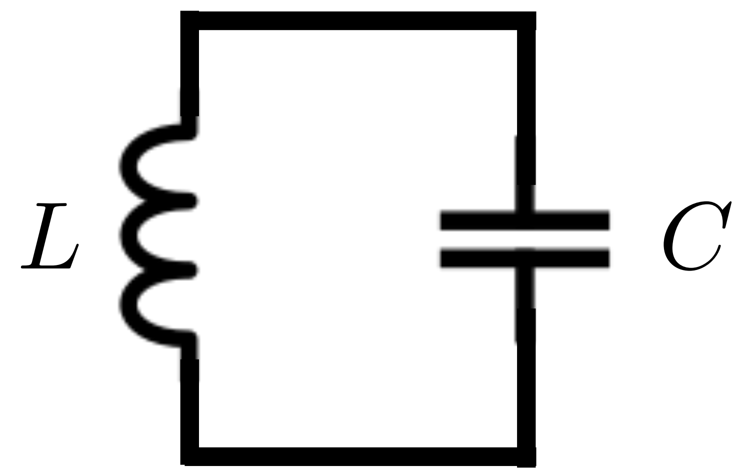


Artificial atoms (superconductors)



Superconducting qubits

Consider a simple oscillating circuit



L inductance C capacitance q electrical charge

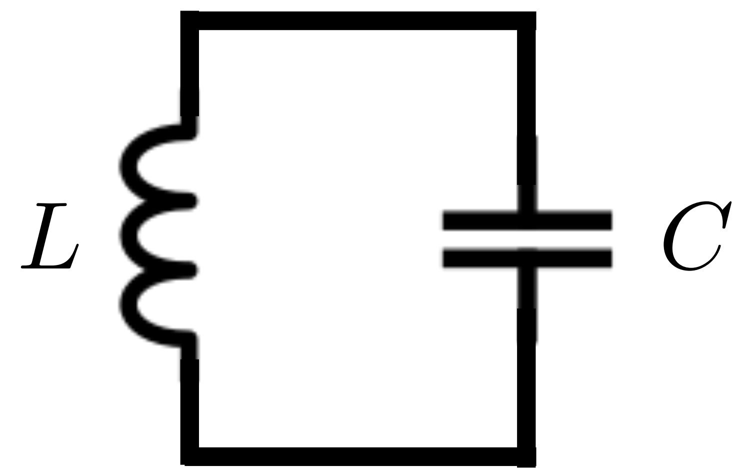
u voltage (potential difference) i electrical current

ϕ magnetic flux (through inductor)

$$i = \frac{dq}{dt} \qquad \phi = L \cdot i \qquad q = C \cdot u \qquad u = -\frac{d\phi}{dt}$$

Superconducting qubits

Consider a simple oscillating circuit



We'll take q and ϕ as our canonical "position" and "momentum"

One can then write the following Hamiltonian

$$H = \frac{q^2}{2C} + \frac{\phi^2}{2L} = \frac{q^2}{2C} + \frac{1}{2}C\omega^2\phi^2$$

$$\omega = \frac{1}{\sqrt{LC}}$$

Superconducting qubits

$$H = \frac{q^2}{2C} + \frac{1}{2}C\omega^2\phi^2$$

This is the same as the Hamiltonian for the harmonic oscillator

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2$$

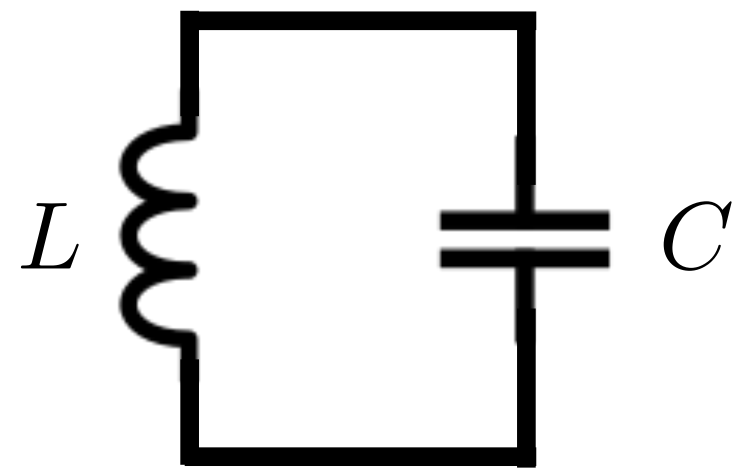
Solving the system entails solving Hamilton's equations

$$\frac{dq}{dt} = -\frac{\partial H}{\partial \phi} \qquad \frac{d\phi}{dt} = \frac{\partial H}{\partial q}$$

$$q(t) = q_0\ cos(\omega t + \alpha_0) \qquad \phi(t) = -\omega q_0\ sin(\omega t + \alpha_0)$$

Superconducting qubits

Consider a simple oscillating circuit



If we cool this circuit to near absolute 0 temperature
it will start to behave quantumly

Classical description no longer applies

But we have canonical quantisation!

Superconducting qubits

$$\hat{H} = \frac{\hat{q}^2}{2C} + \frac{1}{2}C\omega^2\hat{\phi}^2$$

Let's quantise this

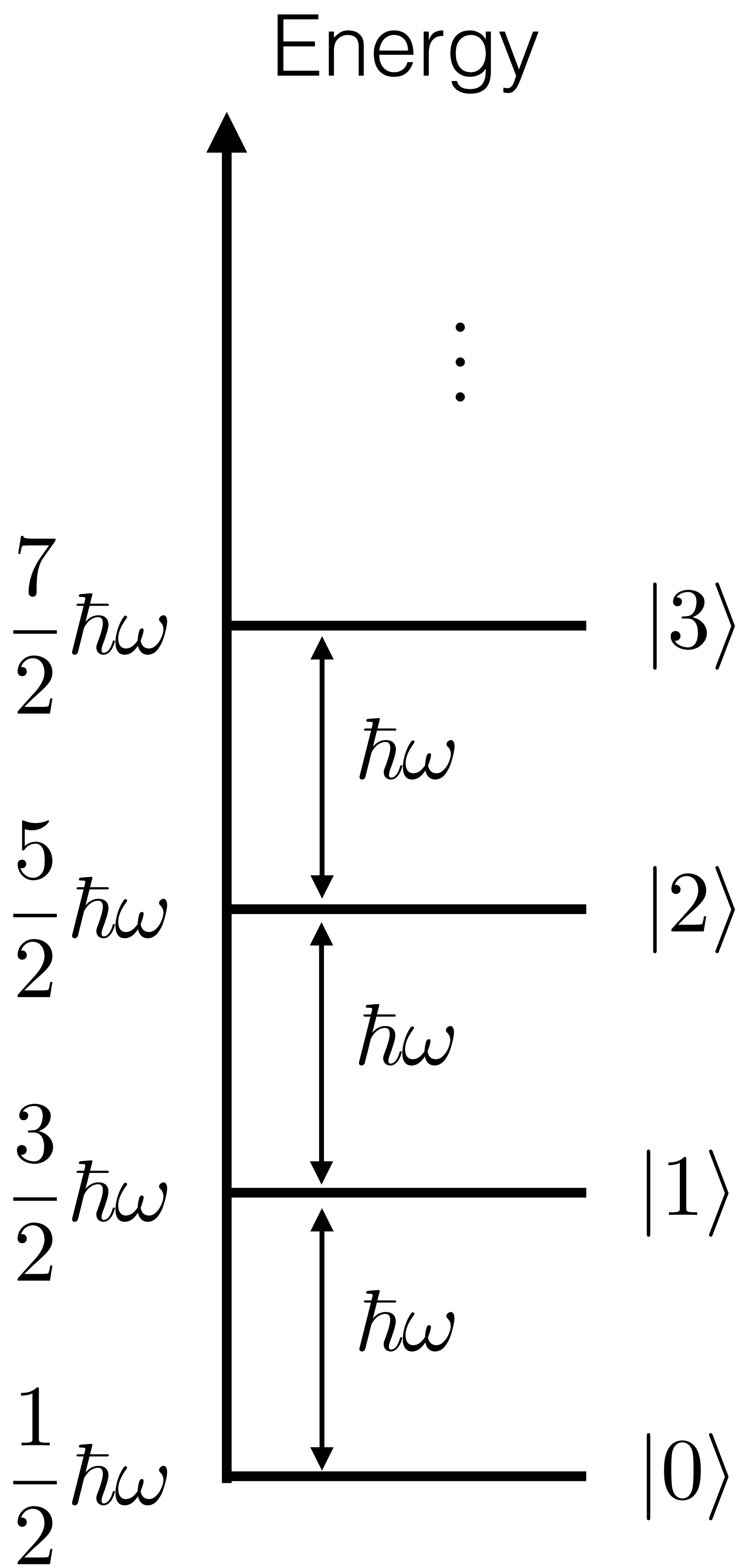
$$[\hat{q}, \hat{\phi}] = i\hbar$$

We want to find the eigenstates of H

There's a standard way of doing this,
which we won't go through :)

What we find is that "neighbouring" eigenstates will
be separated by the same energy

Superconducting qubits



$$\omega = \frac{1}{\sqrt{LC}}$$

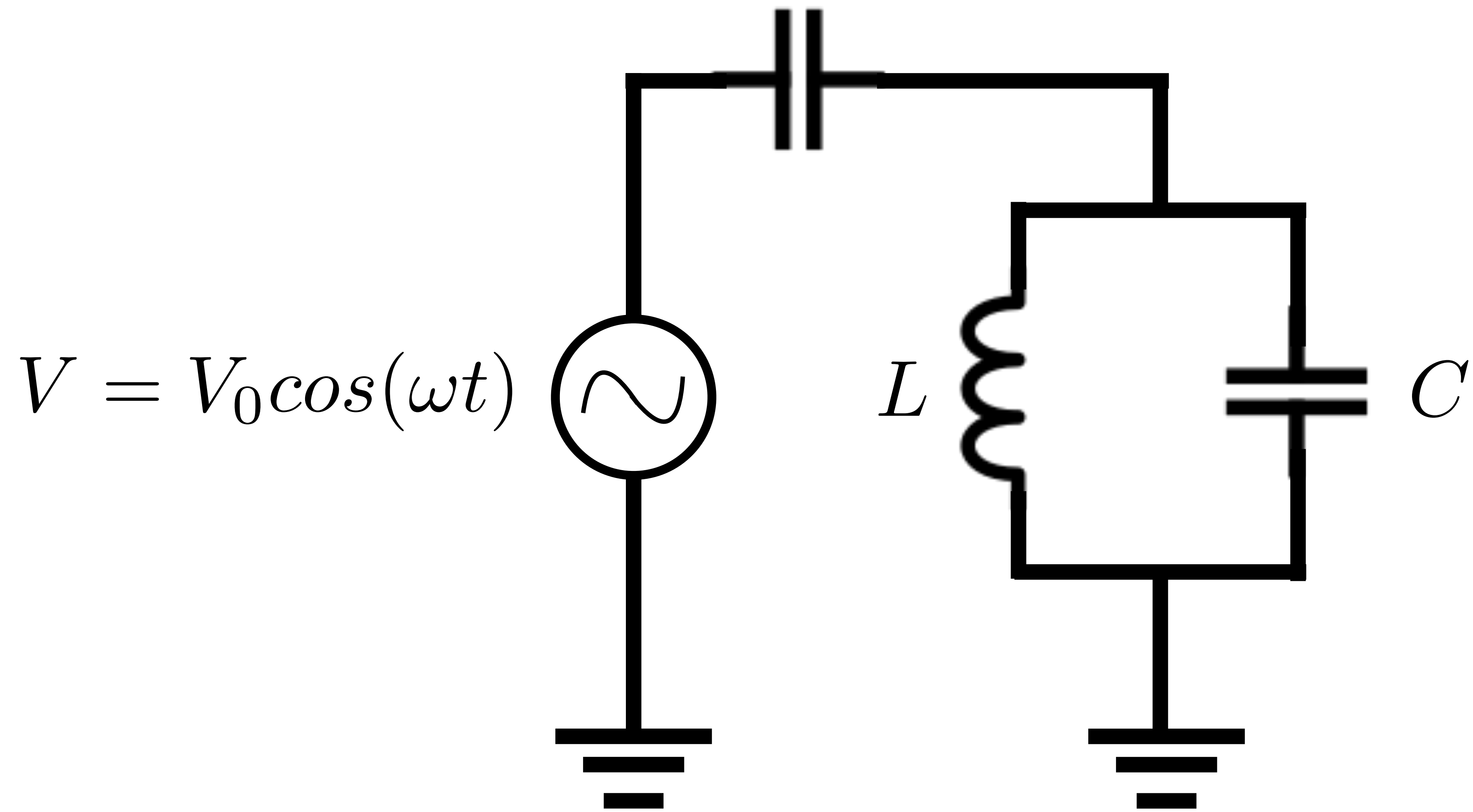
Energy levels are equally spaced

This is unlike atoms!

How do we excite the system?

With EM radiation of course!

Superconducting qubits



Typical values for L and C $\quad L \approx 1nH \quad C \approx 1pF$

$$\omega \approx 10 - 30GHz$$

Microwaves!

Superconducting qubits

But there's a problem!

A laser or our microwave source do not produce single-photon states

Instead, they produce **coherent states**

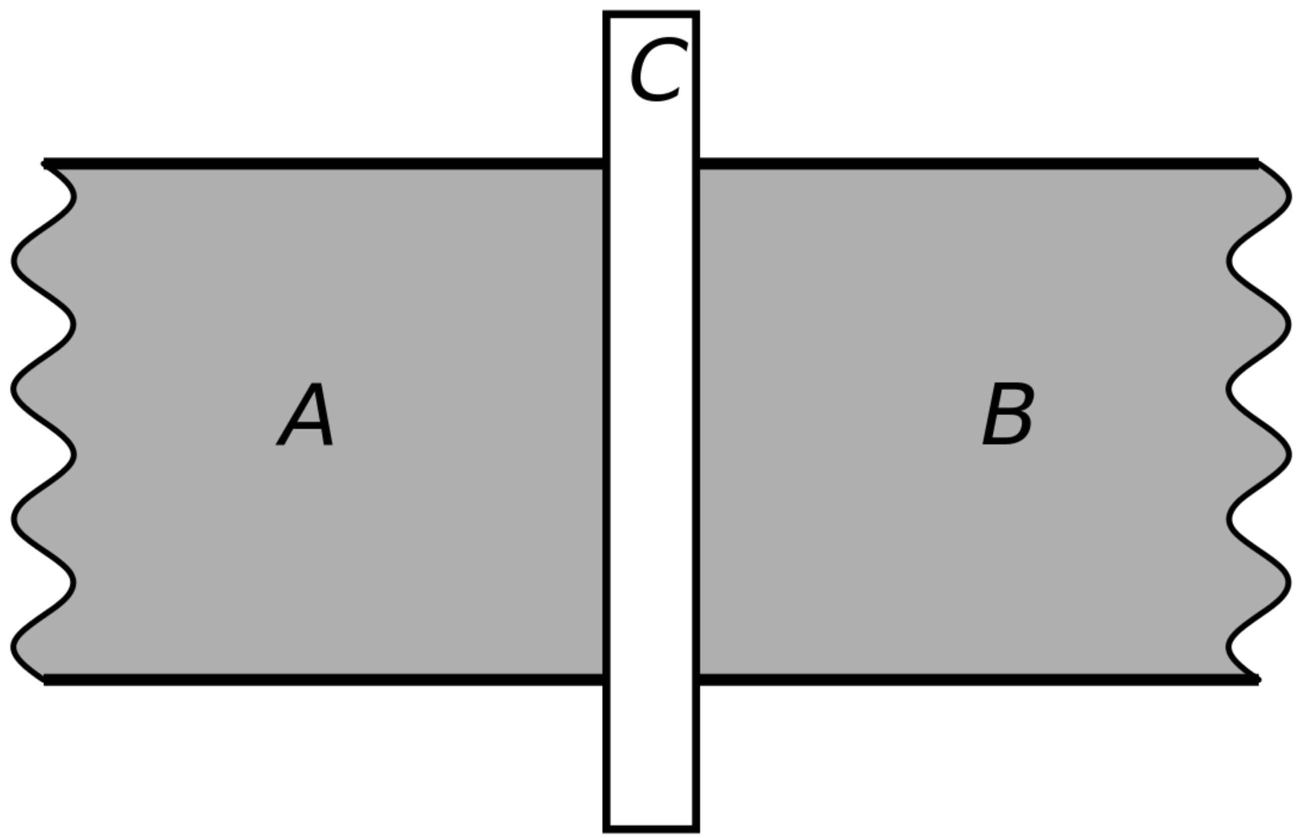
$$|\psi\rangle = e^{-\frac{|\alpha|^2}{2}}\left(|0\rangle + \alpha|1\rangle + \frac{\alpha^2}{\sqrt{2}}|2\rangle + \frac{\alpha^3}{\sqrt{3}}|3\rangle + \ldots\right.$$

This means that our "qubit" will be in a superposition of all energy levels!

Superconducting qubits

We need to "space out" the energy levels

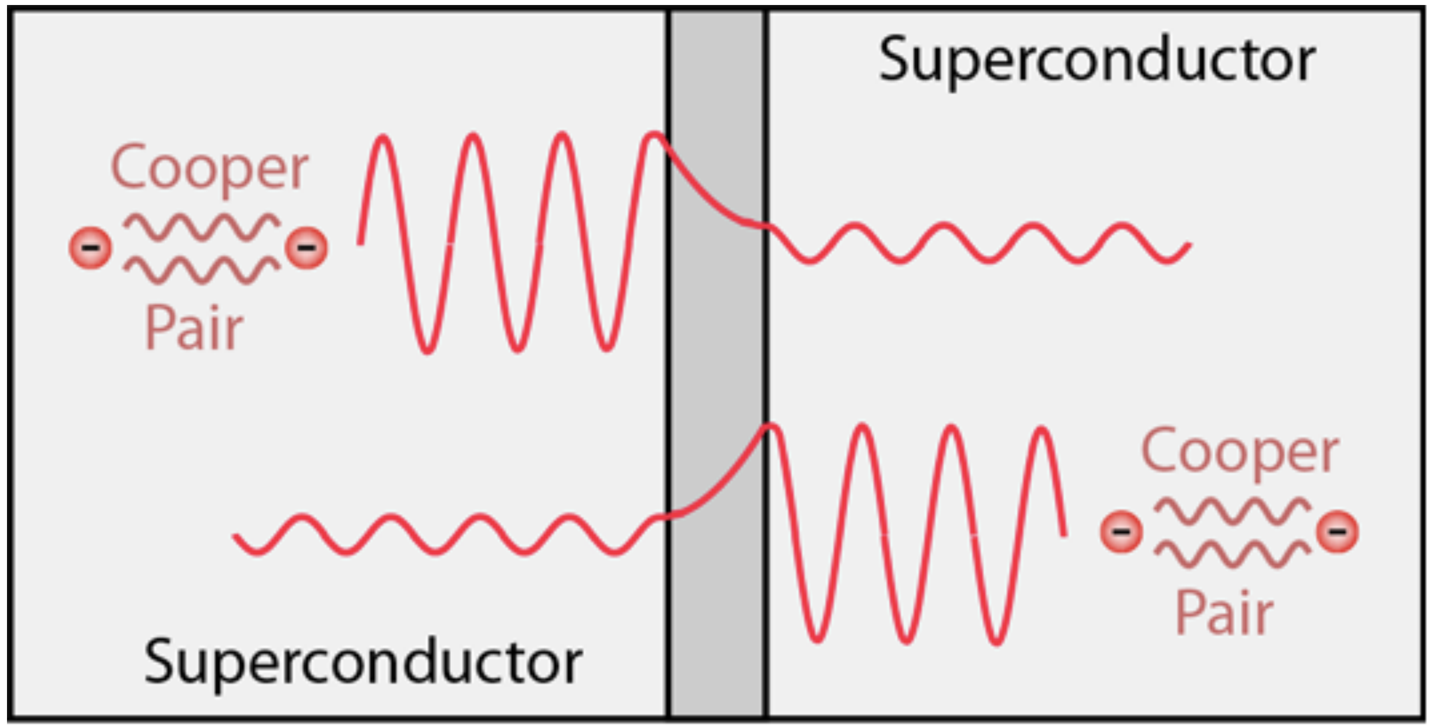
Josephson junction



A and B regions are superconductors

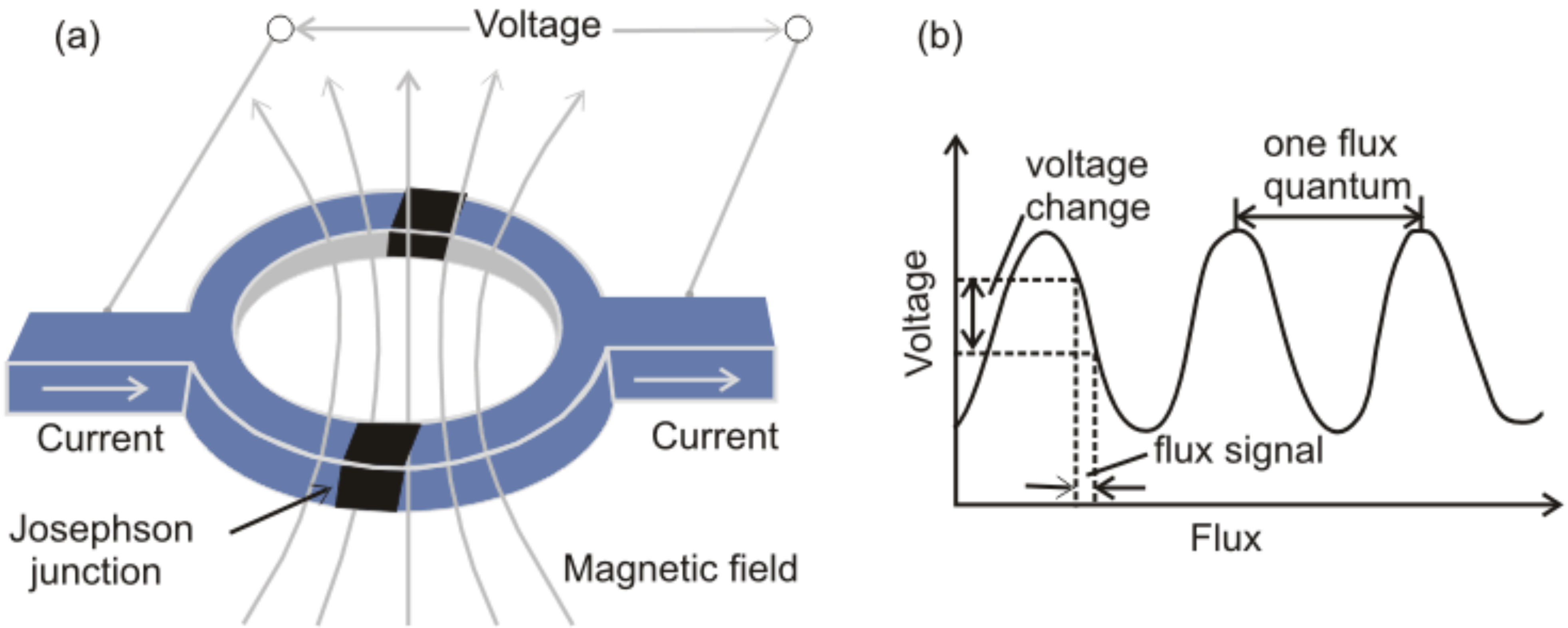
C region is an insulator

Superconducting qubits



Current can tunnel through the insulator

Superconducting qubits



Current can tunnel through the insulator

The junction acts as a non-linear inductor

$$i = f(\phi)$$

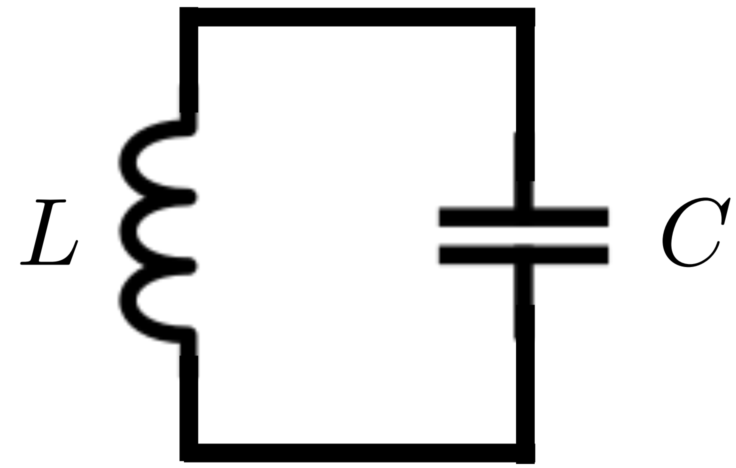
Inductor

$$i = \phi/L$$

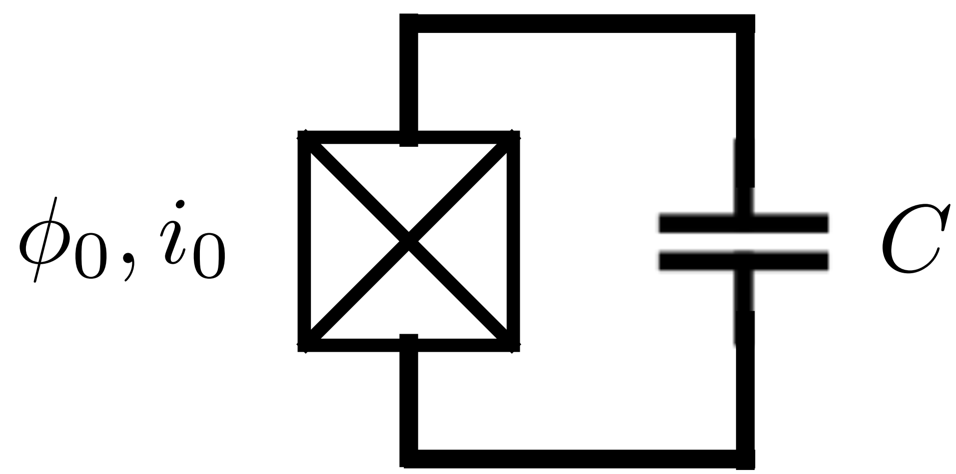
Josephson junction

$$i = i_0 sin(2\pi\phi/\phi_0)$$

Superconducting qubits

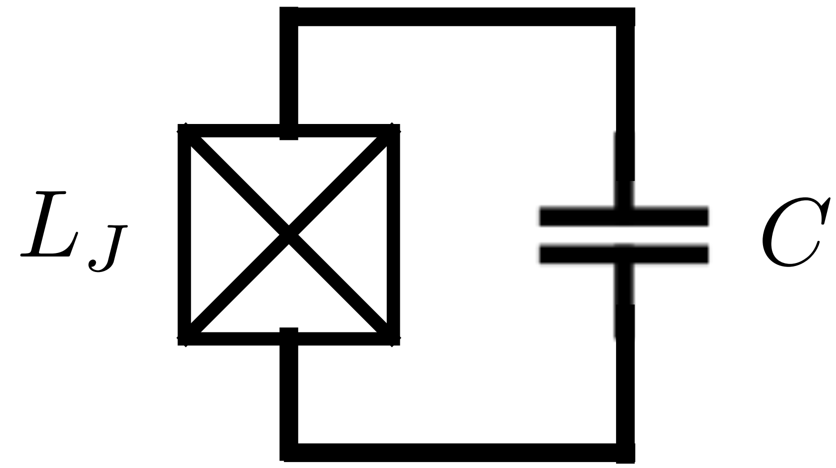


Is replaced with



We can still treat the Josephson junction as an inductor

Superconducting qubits

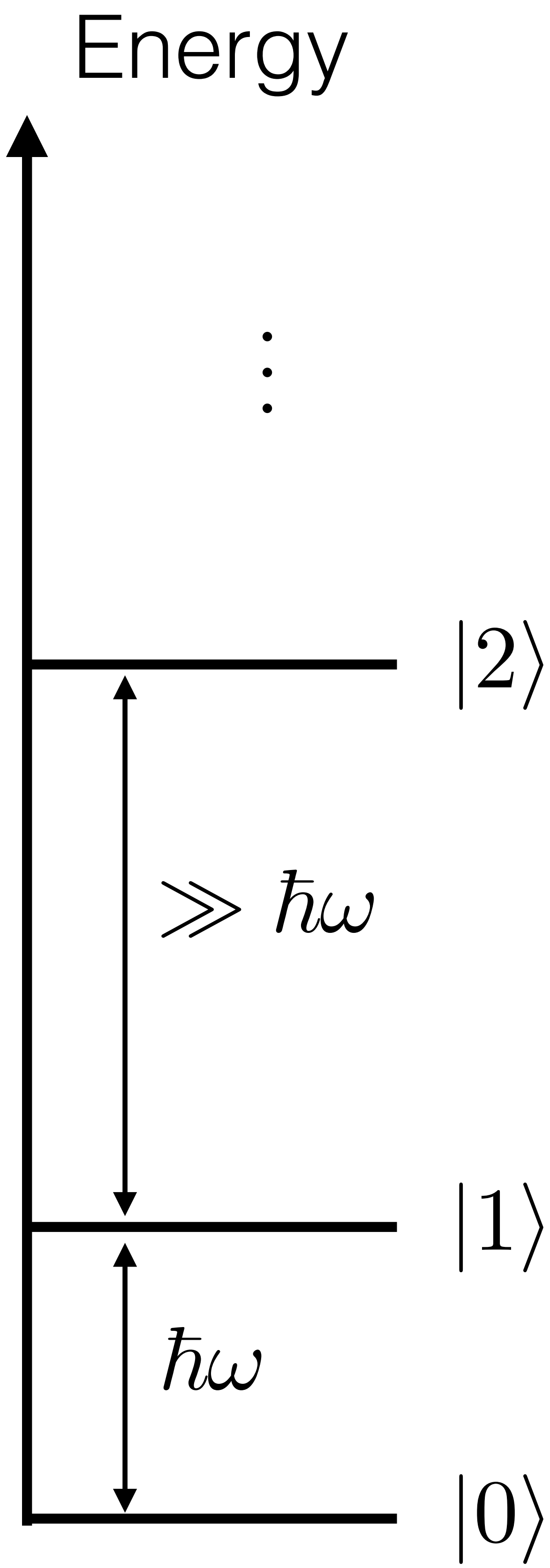


$$L_J = \left(\frac{\partial i}{\partial \phi}\right)^{-1}$$

$$L_J = \frac{\phi_0}{2\pi i_0}\frac{1}{cos(2\pi\phi/\phi_0)}$$

$$H = \frac{q^2}{2C} + \frac{\phi^2}{2L_J(\phi)}$$

Superconducting qubits



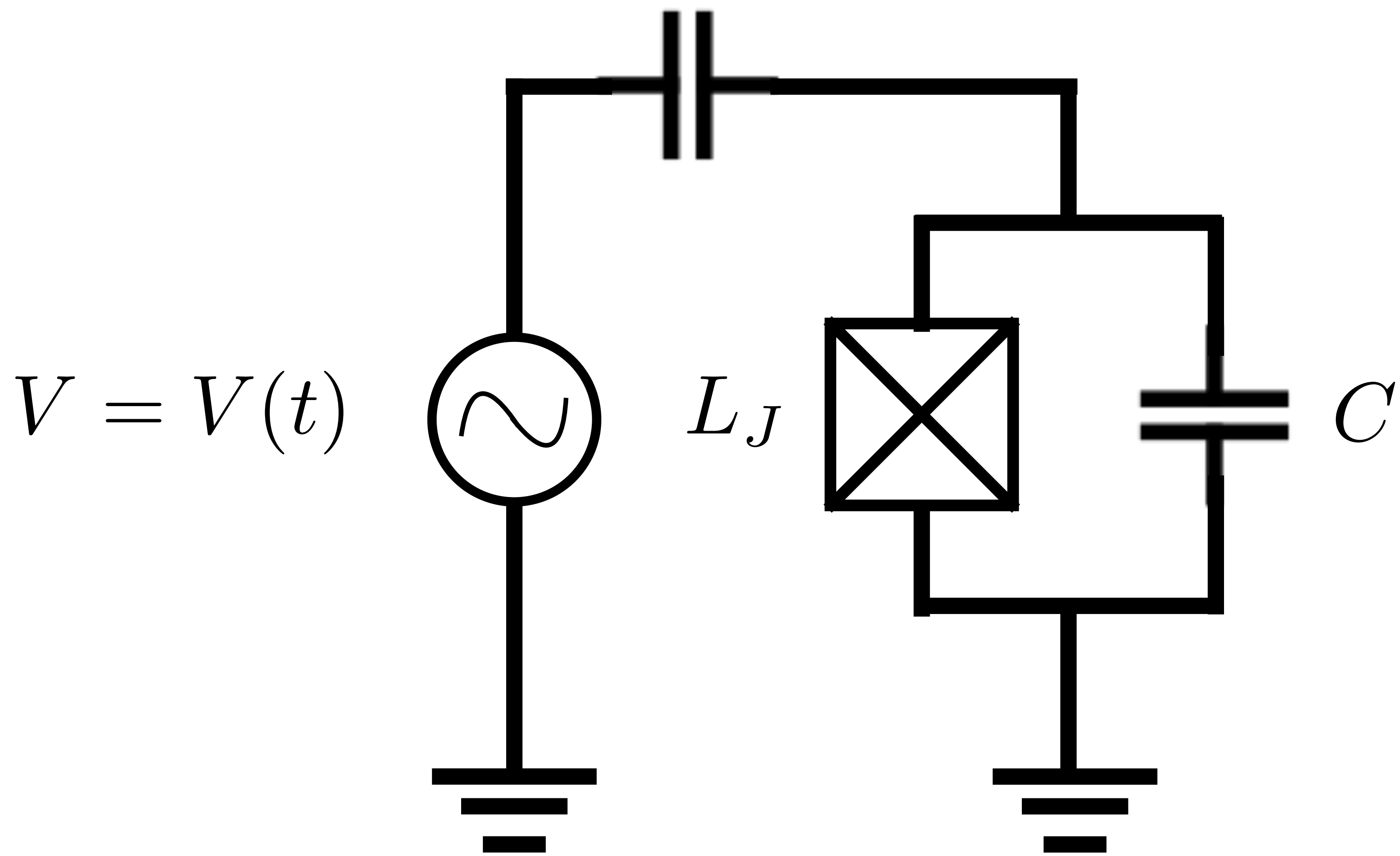
This is effectively at 2-level system

Some typical values for the critical current and Josephson inductance

$$i_0 \approx 100nA \qquad L_J \approx 3nH$$

So how do we do single-qubit gates?

Superconducting qubits



$$V(t) = V_x(t)cos(\omega_d t) + V_y(t)sin(\omega_d t)$$

$$H_d = (\omega - \omega_d)|1\rangle\langle 1| + \frac{V_x(t)}{2}X + \frac{V_y(t)}{2}Y$$

ω_d is the *driving frequency*

Superconducting qubits

The true Hamiltonian of our system will be

$$H = \frac{q^2}{2C} + \frac{\phi^2}{2L_J(\phi)} + H_d$$

We can assume that the system starts out in the ground state of the original Hamiltonian

The driving Hamiltonian will change this state

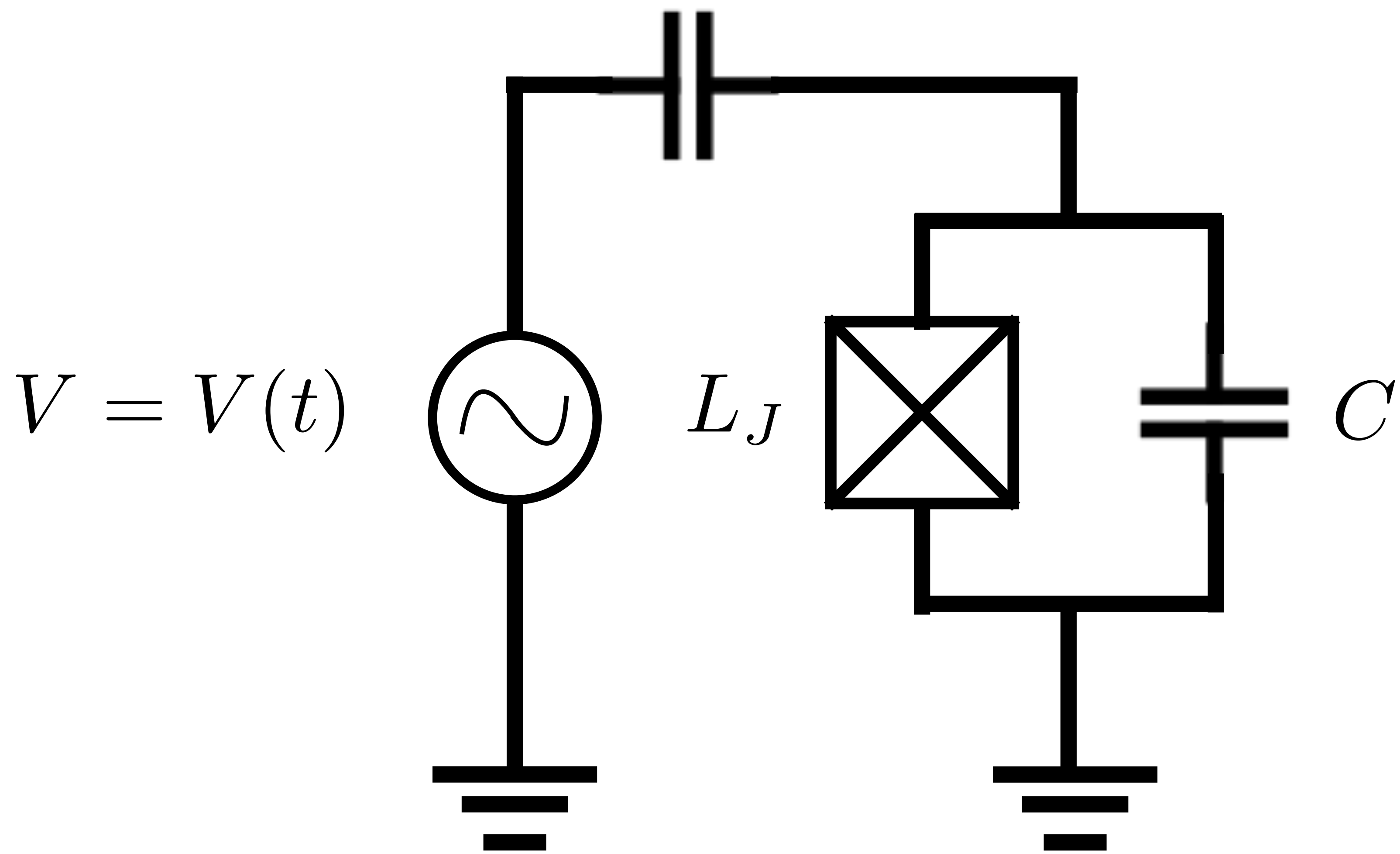
$$|0\rangle \rightarrow_{H_d} U|0\rangle$$

$$U = exp\Big\{ -\frac{i}{\hbar}\int_0^{t_d} H_d dt \Big\}$$

Superconducting qubits

To sum up

Prepare this circuit

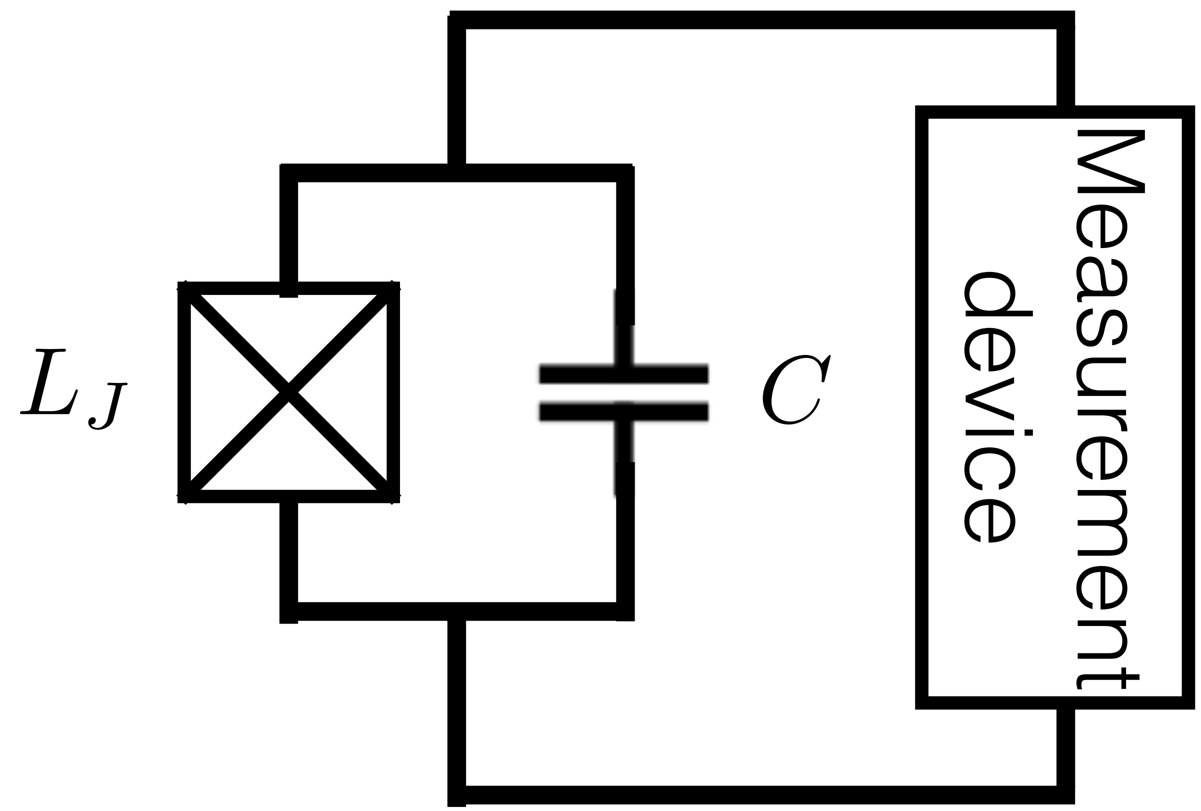


Choose what unitary you'd like to perform

Apply microwave radiation of the right type and for the appropriate time

Superconducting qubits

Measurement

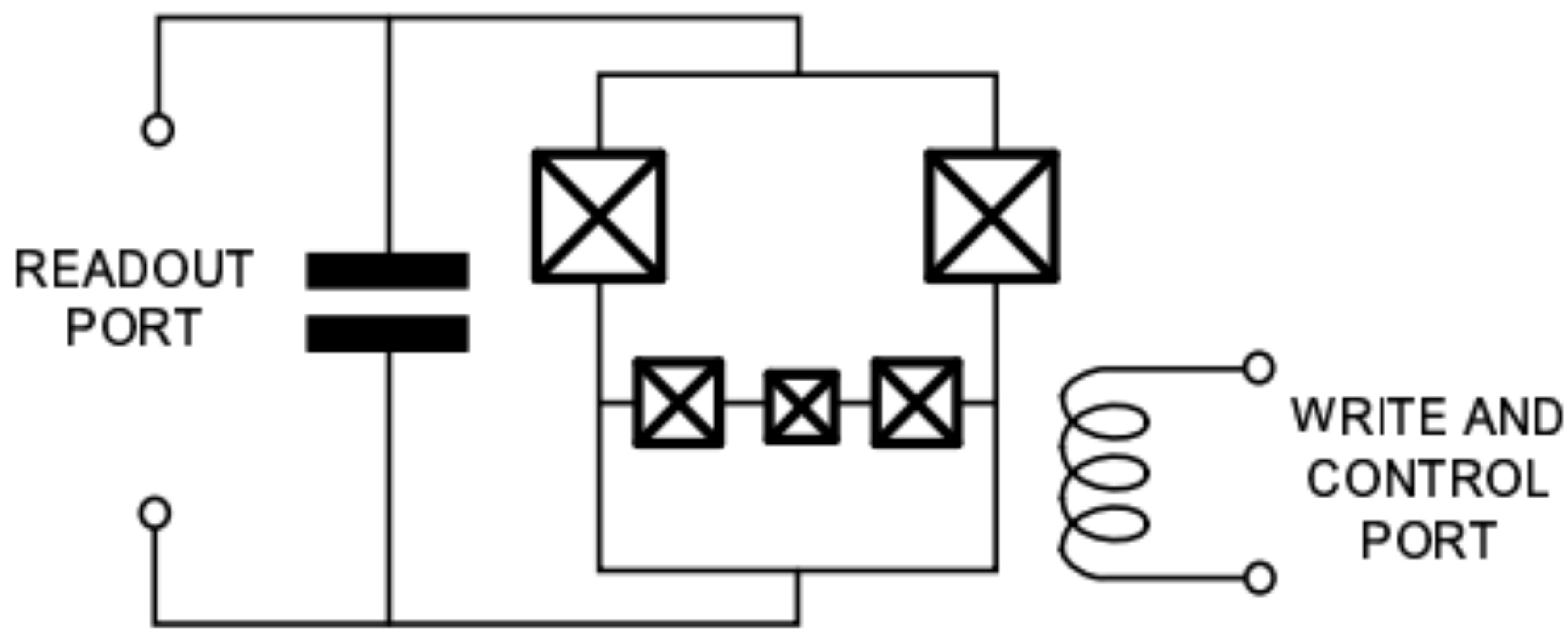


This is very “schematic”

What we measure depends on the type of qubit we have (flux qubit, charge qubit, phase qubit)

Superconducting qubits

Other configurations are also possible

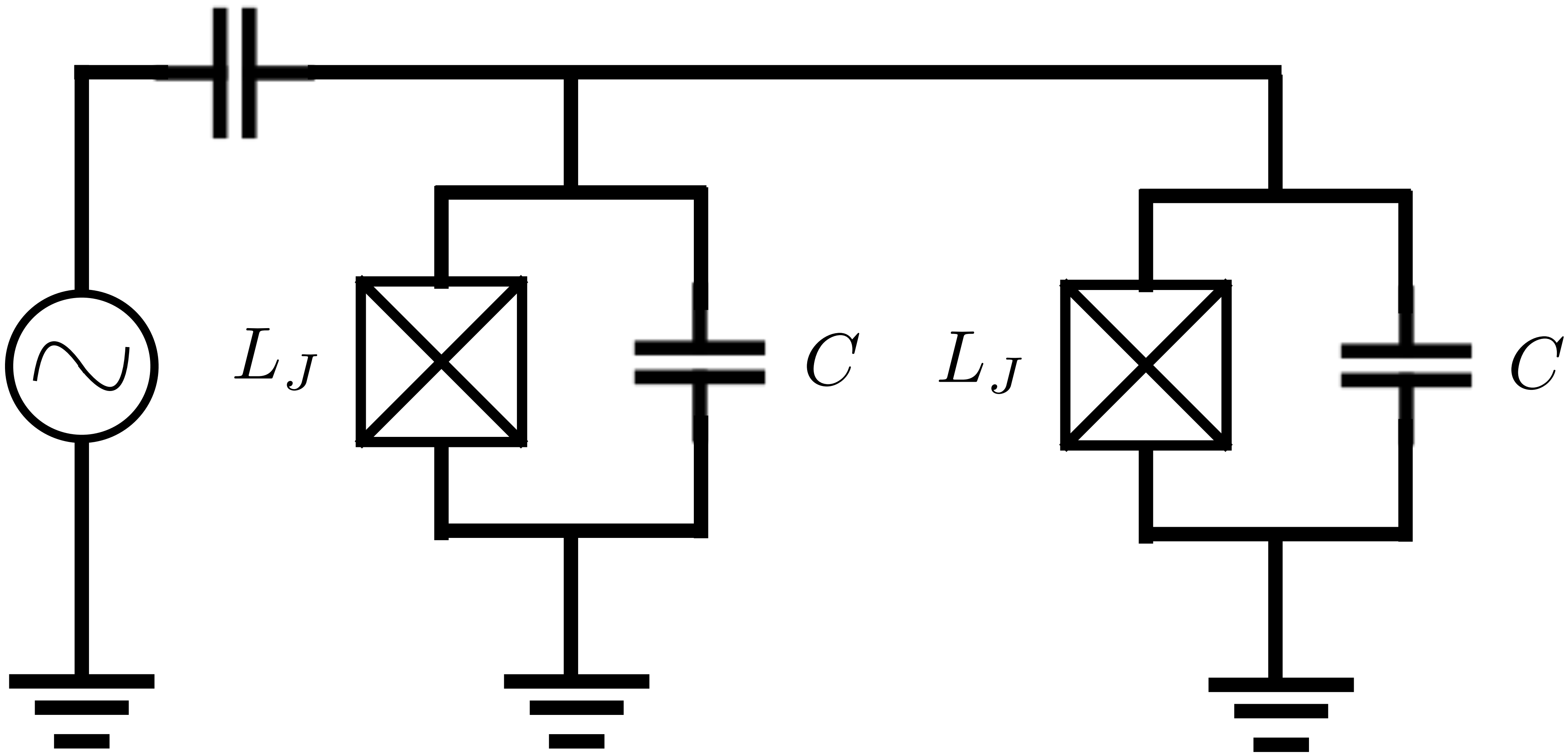


SQUIDs, fluxonium, transmon, xmon, quantronium

Physicists have the better names again :)

Superconducting qubits

Multiple qubits



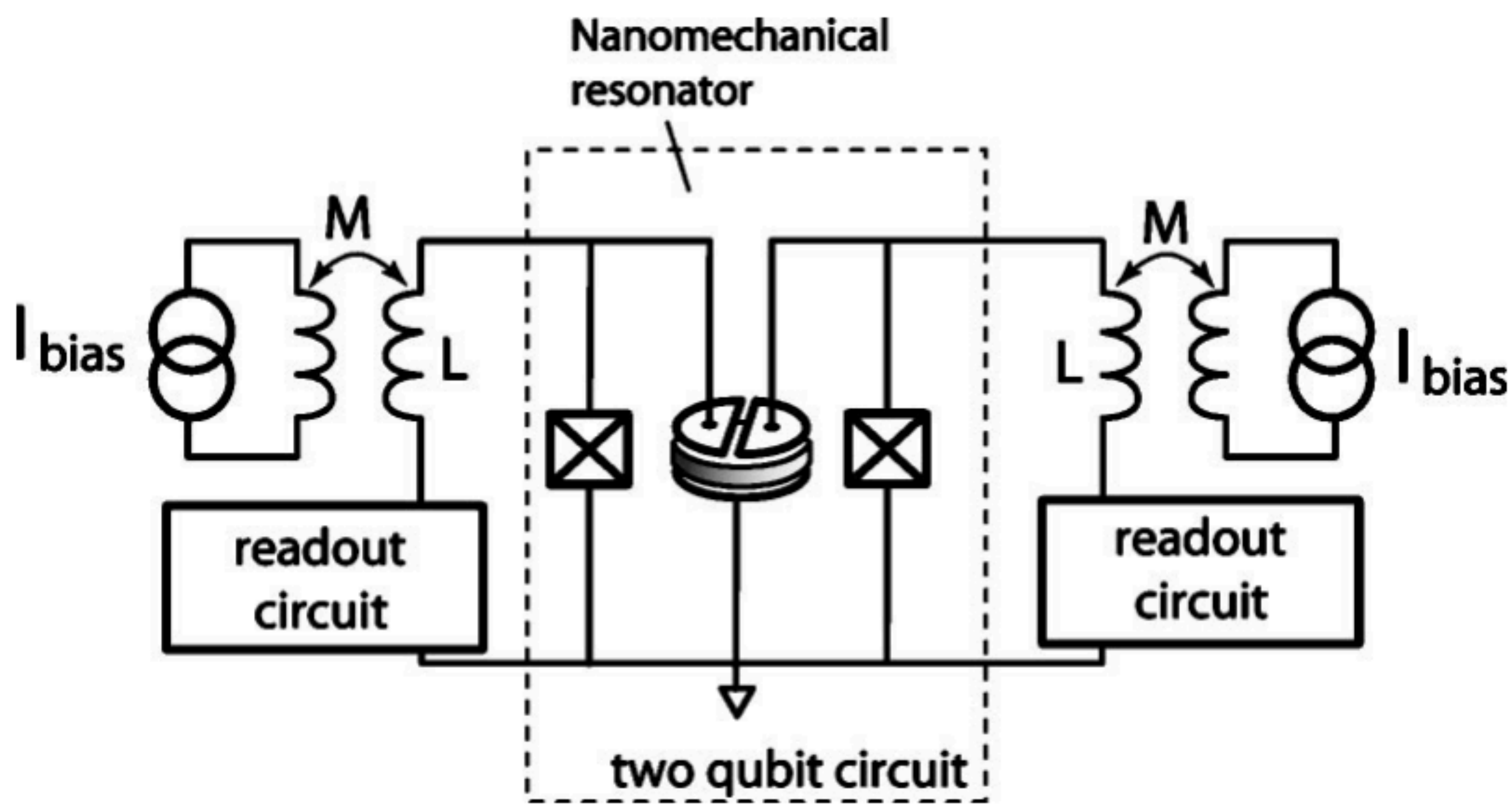
H_d will have interaction terms of the form $X \otimes X, Y \otimes Y$

Superconducting qubits

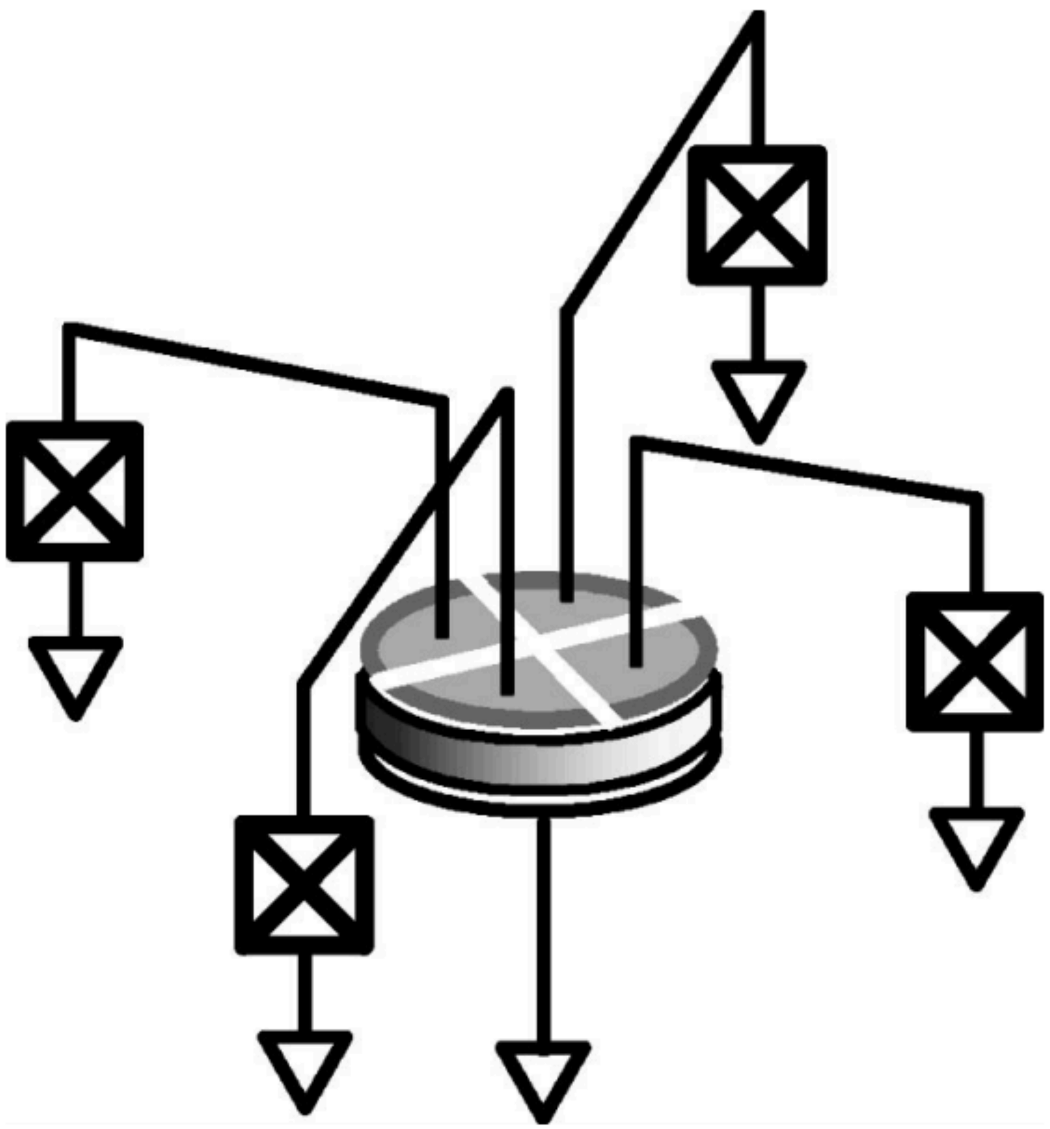
$$exp(iX \otimes Xt)$$

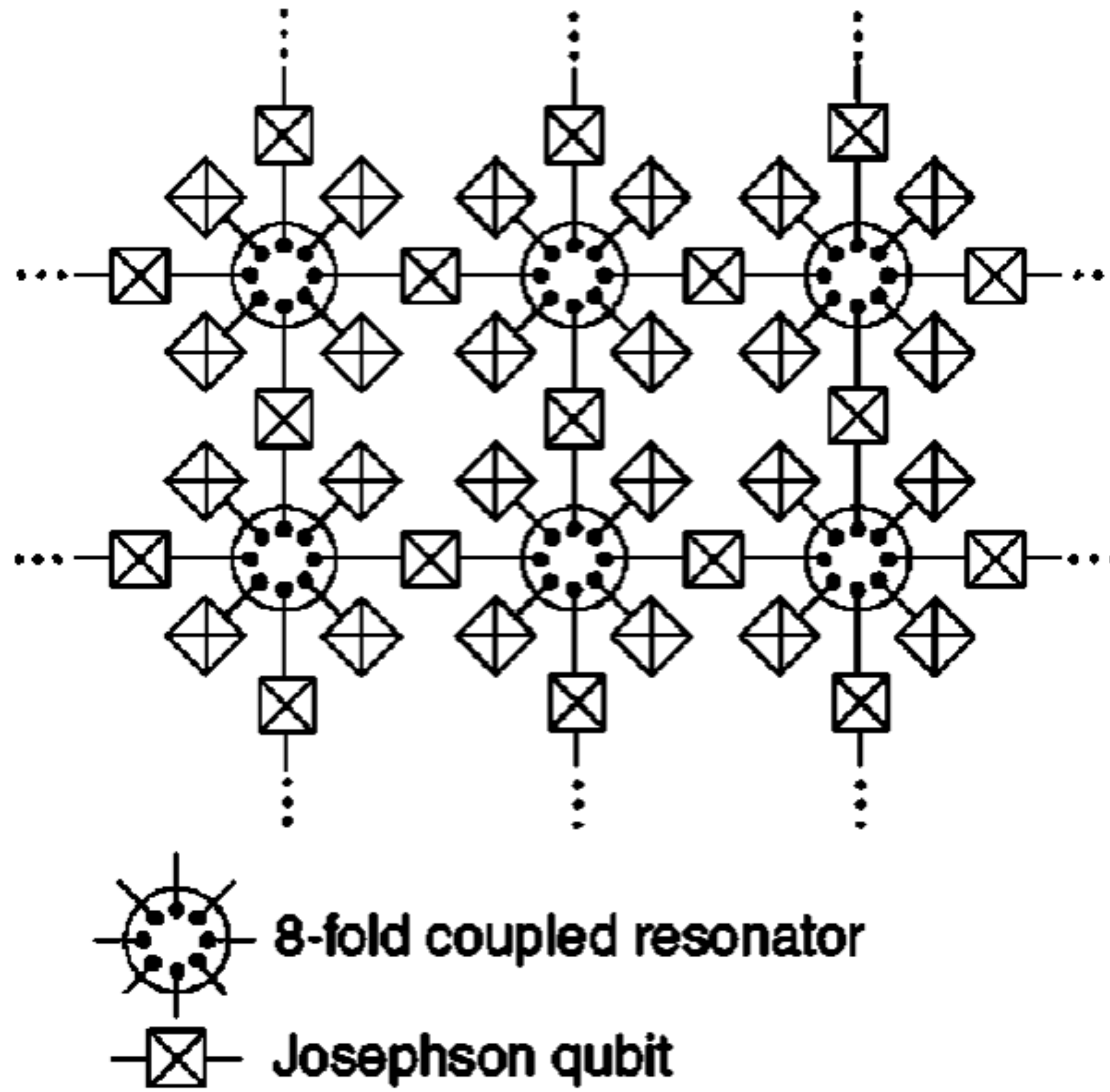
together with local unitaries is universal

Again, this is schematic. In practice other configurations



Superconducting qubits





Superconducting qubits

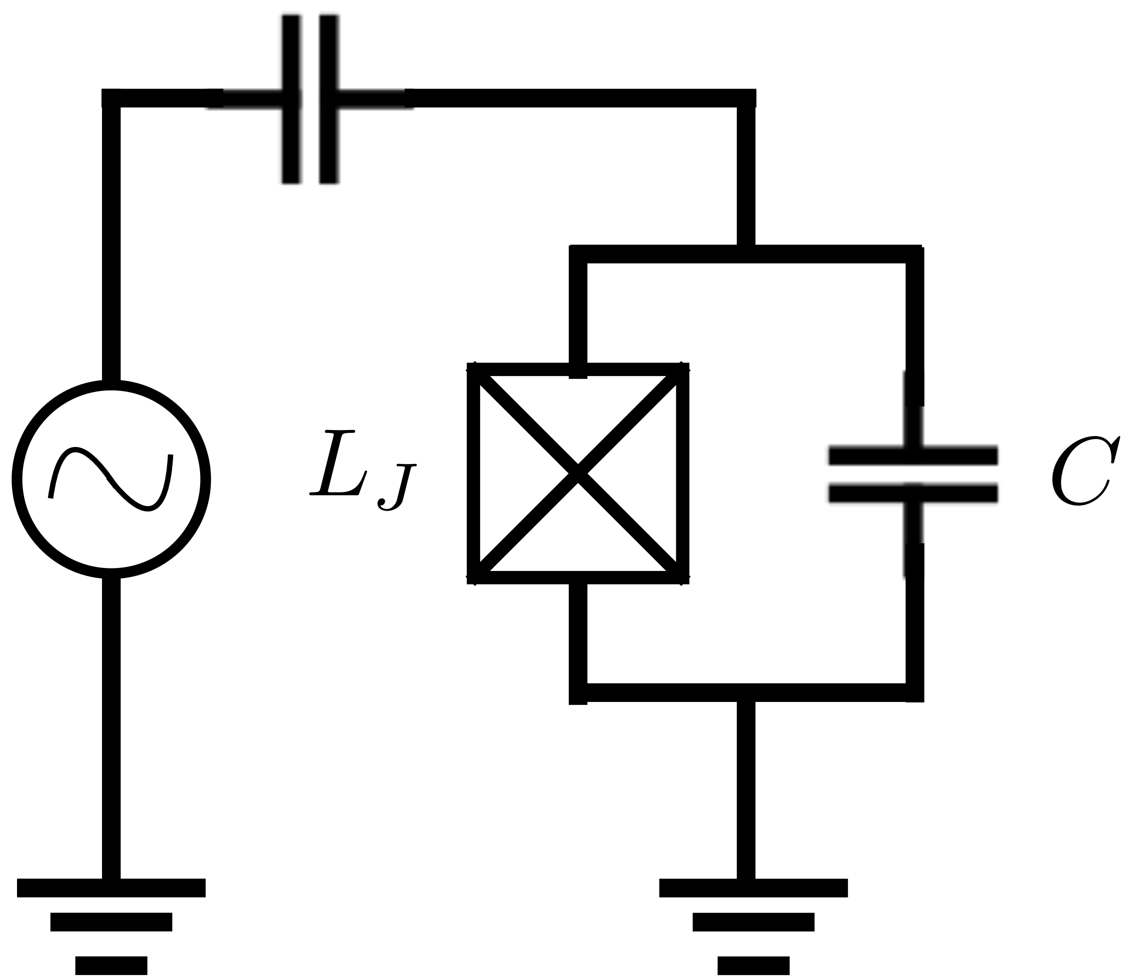
Error rates for different implementations

	Single qubit	Multi-qubit	Readout
IBM	0.1%	1%	1%
Rigetti	$2-5\%$	$7-18\%$	$8-20\%$
Google	$0.05-0.1\%$	$0.5-1\%$	$0.5-1\%$

But how stable are these qubits?

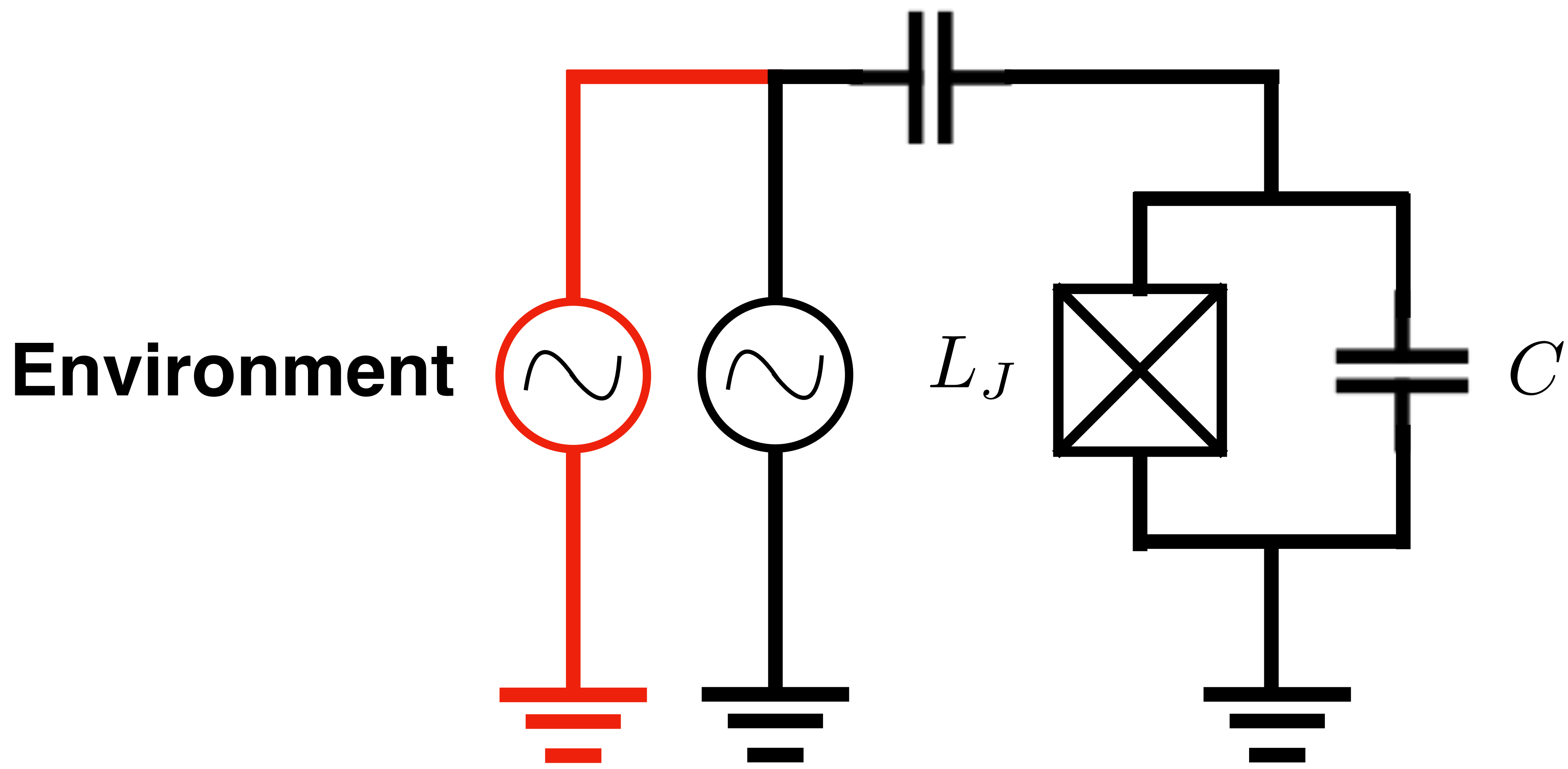
Superconducting qubits

Ideally



Superconducting qubits

In reality



Noise from the environment can corrupt our qubits

Radiation, heat, vibrations etc

Superconducting qubits

Initially I prepare

$$|\psi\rangle = a|0\rangle + b|1\rangle$$

After some time, t

91%	3%	3%	3%
$\|\psi\rangle$	$X\|\psi\rangle$	$Z\|\psi\rangle$	$XZ\|\psi\rangle$

After more time, t

82%	6%	6%	6%
$\|\psi\rangle$	$X\|\psi\rangle$	$Z\|\psi\rangle$	$XZ\|\psi\rangle$

Superconducting qubits

This is called **decoherence**

More on this in the next lecture

$$|\psi\rangle = a|0\rangle + b|1\rangle$$

T_d average time for this ↓ **Decoherence time**

25%	25%	25%	25%
$\|\psi\rangle$	$X\|\psi\rangle$	$Z\|\psi\rangle$	$XZ\|\psi\rangle$

Also called **coherence time** :)

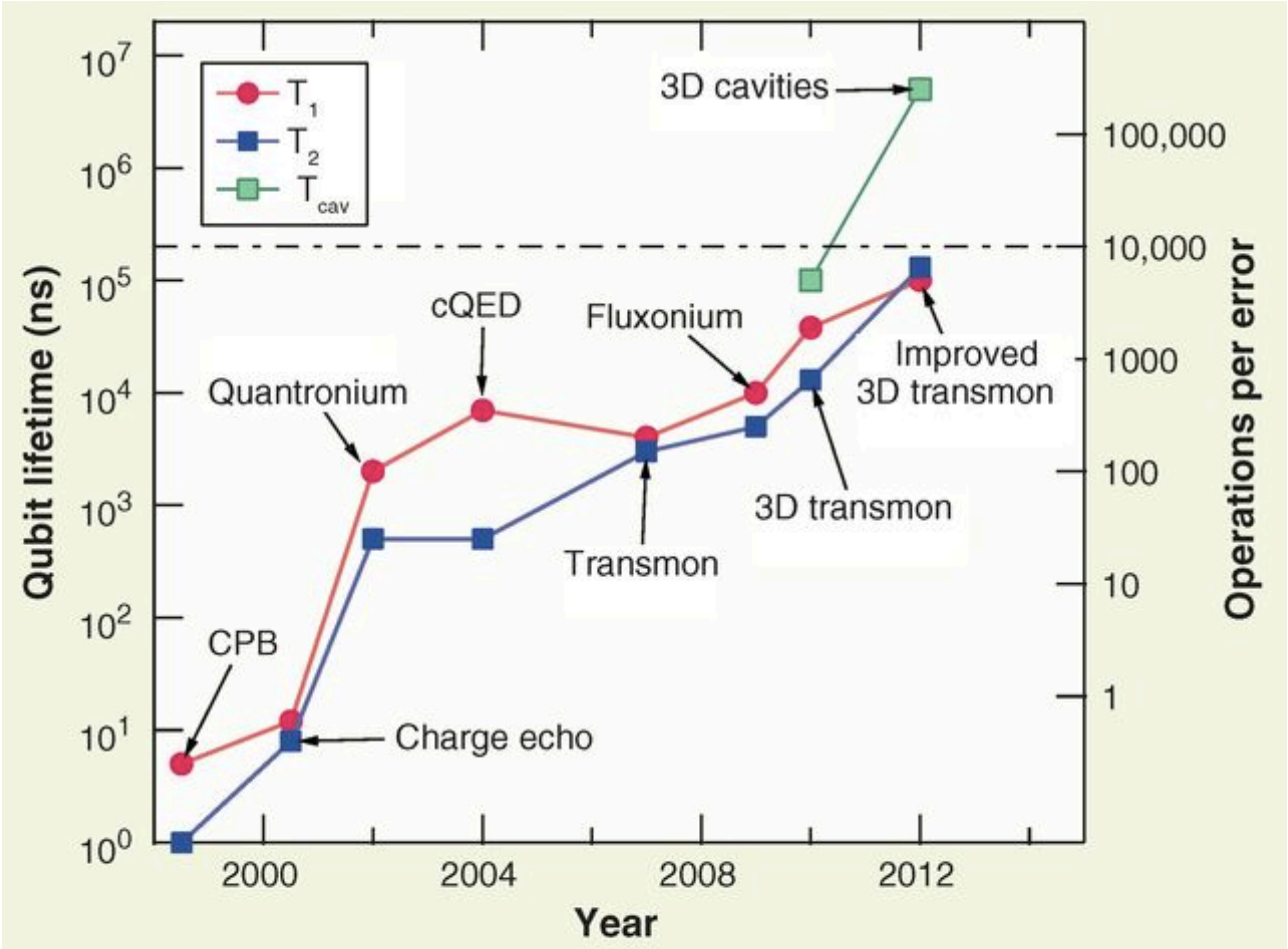
Superconducting qubits

	Coherence/decoherence times
IBM	$50-77\mu s$
Rigetti	$10-26\mu s$
Google	$\approx 50\mu s$
D-Wave	$\approx 100ns$

Time to perform a quantum operation (for IBM, Rigetti, Google)

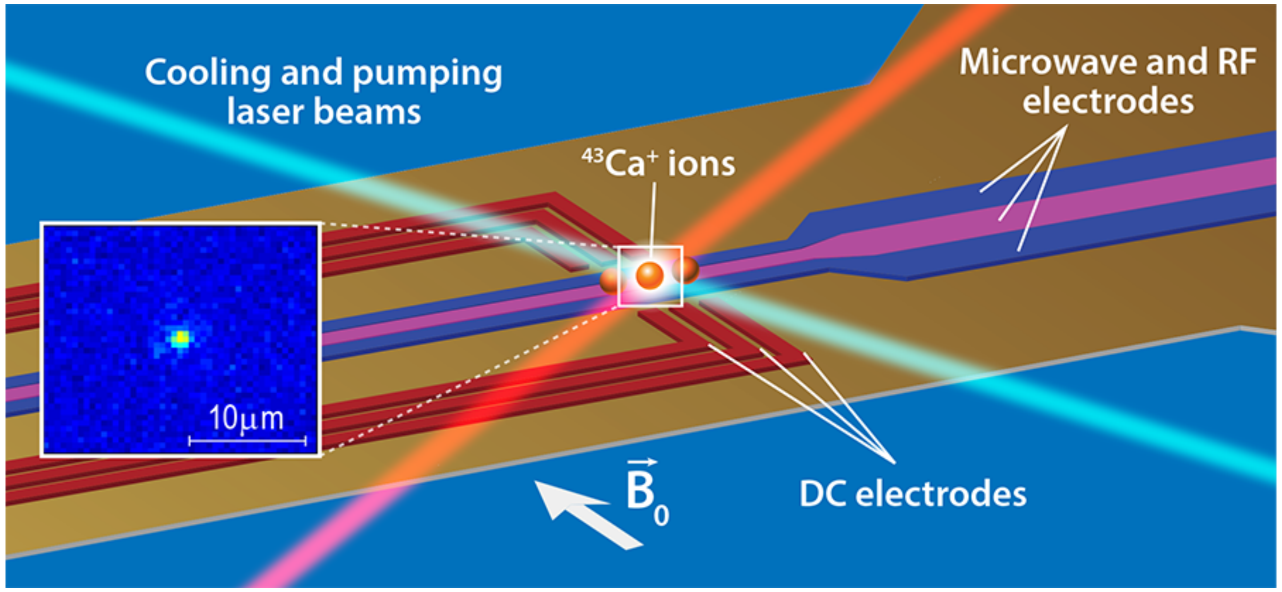
$$10-100ns$$

Superconducting qubits



http://science.sciencemag.org/content/339/6124/1169/F3

Ion traps



Same idea as before, but using nature's atoms

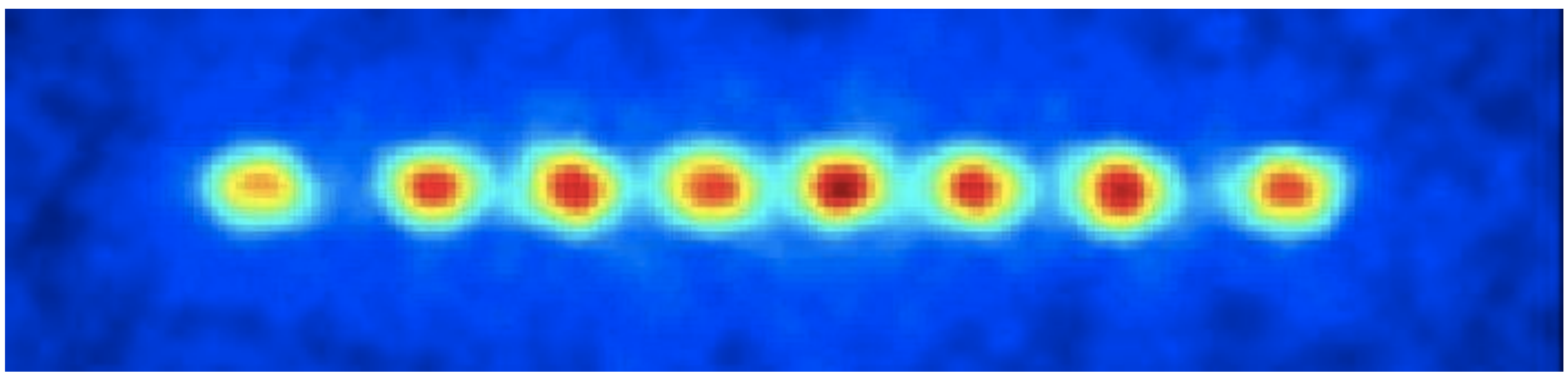
We need to capture them, hence why we use ions

This can be done with a combination of electric and magnetic fields (Penning trap, Paul trap etc)

Ion traps

Of course, we need more than just one ion!

Linear ion traps

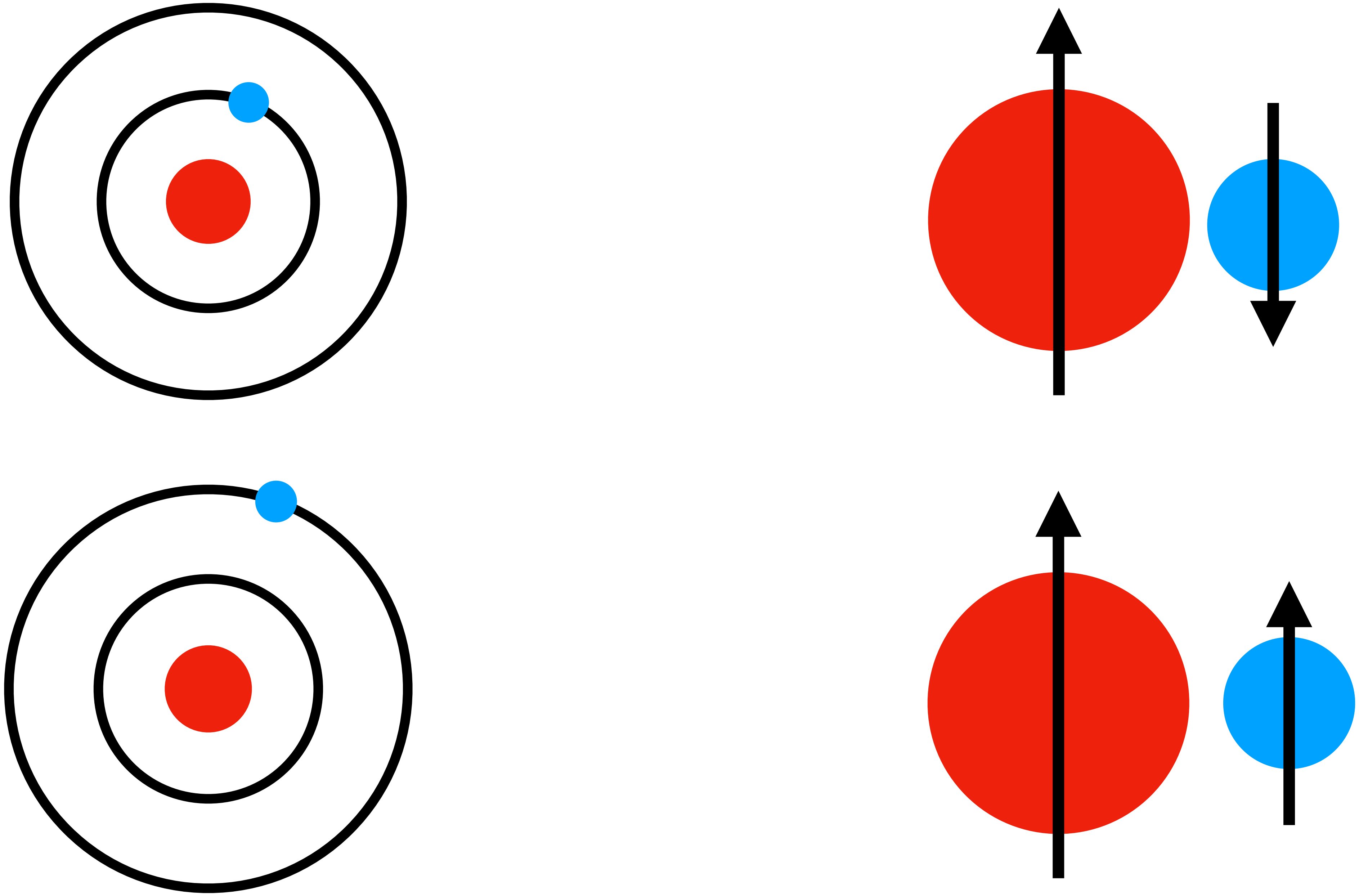


Typically the ions are kept tens of micrometers apart

Enough so that Coulomb interaction becomes important!

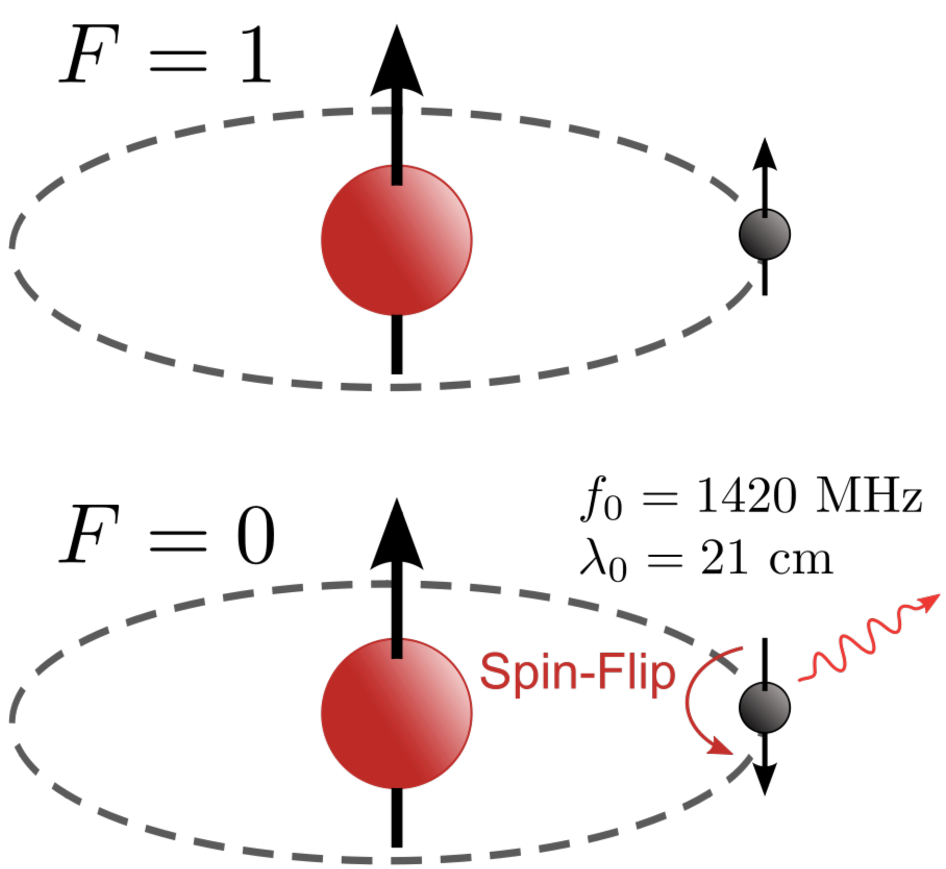
Ion traps

Either use energy levels (optical qubits)
or hyperfine energy levels (hyperfine qubits)



Ion traps

The 21cm line of hydrogen

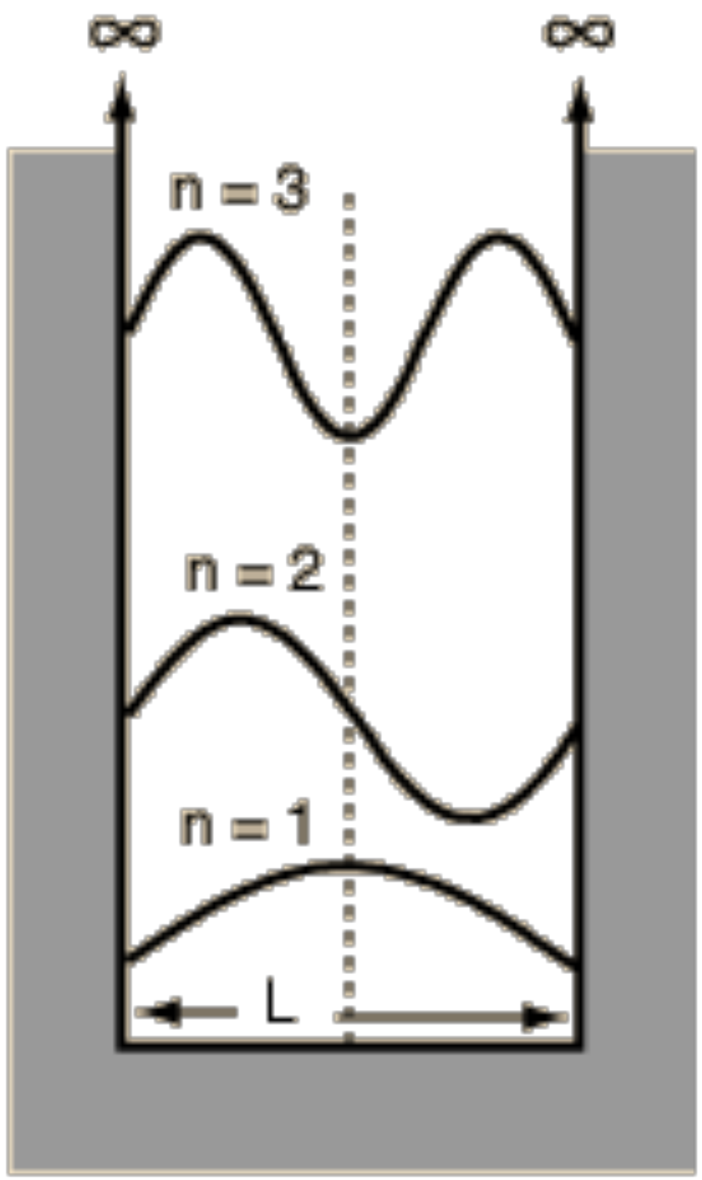


Hyperfine levels are very stable and have have long decay times

"Regular" levels have short decay times (spontaneus emission/decay)

Ion traps

The position of the ions is also quantised!



x = 0 at left wall of box.

Thus, the state of each ion is

$$|m, n\rangle$$

Energy state

Position state

Importantly, because the ions repel each other, their positions can become entangled

Ion traps

In fact, we quantise the position of <u>all ions together</u>

The basis states of the whole system of k ions will be

$$|m_1, m_2, ...m_k, n\rangle$$

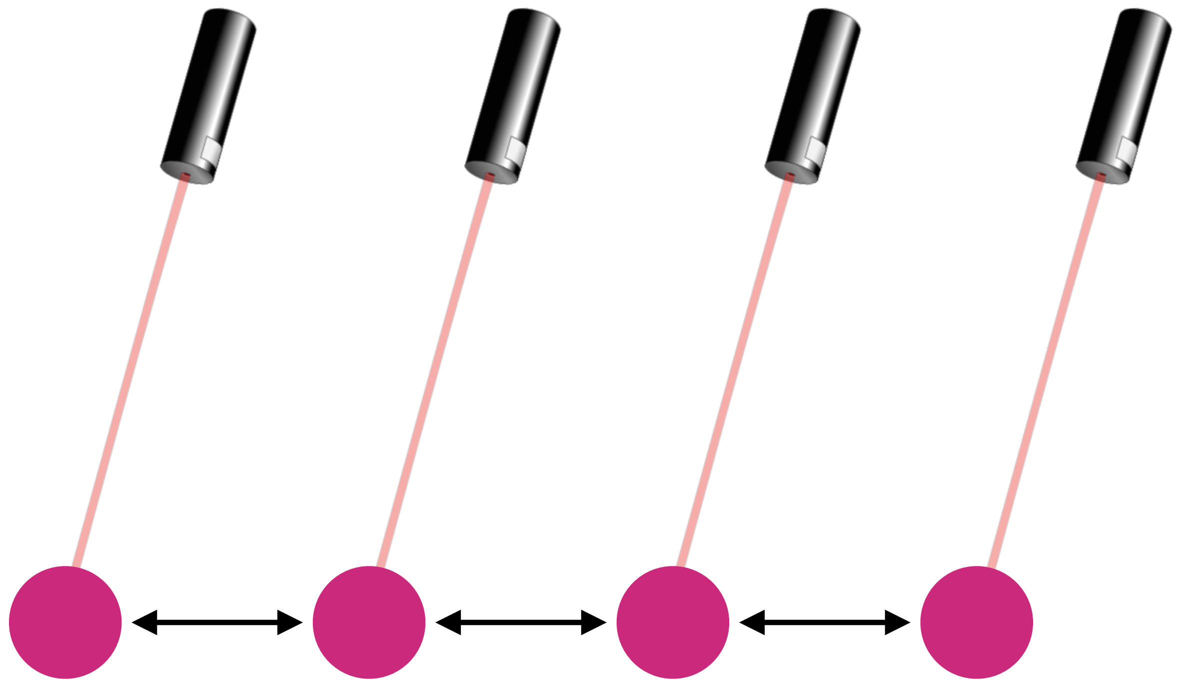
The m's label the energy states of each ion

n labels the position states of all ions

We also say that we have n **phonons** in the system

Quantised vibrations of the ions

Ion traps



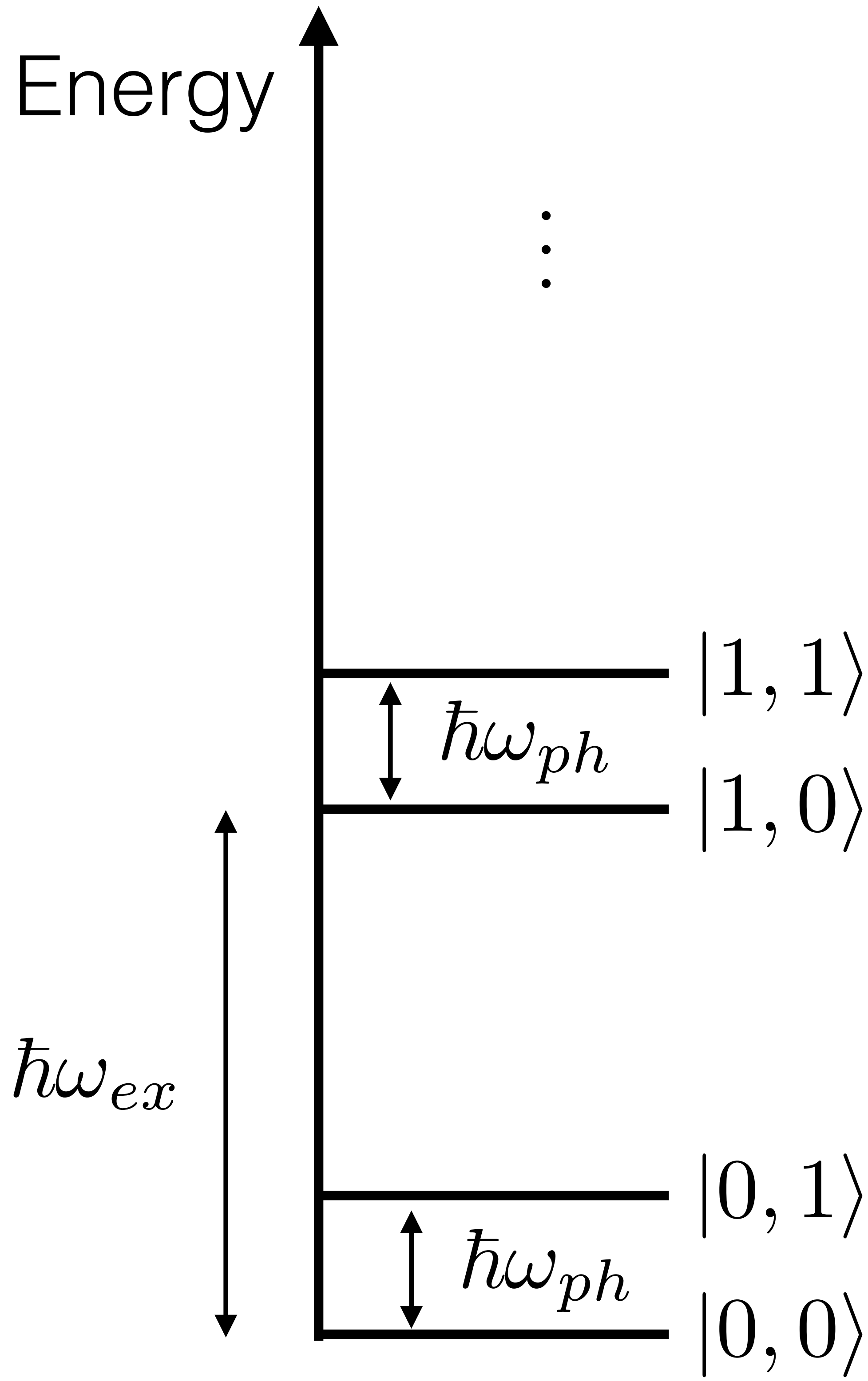
Depending on the type of ion qubits we have, changing their state is done with either lasers or RF radiation

Laser can be used to perform single-qubit gates on one ion

But also 2-qubit gates by entangling its energy state to the position state!

Ion traps

Let's look at one ion



Shine a laser of frequency

$$\omega = \omega_{ex} + \omega_{ph}$$

For some time t

$$|0,0\rangle \rightarrow a(t)|0,0\rangle + b(t)|1,1\rangle$$

This will entangle the ion's energy state to the phonon modes

(technically, we might want to use a 3-level atom)

Ion traps

Entangle ion to phonon modes

Entangle another ion to phonon modes

We've entangled 2 ions!

With frequencies around ω_{ex} we can do single qubit operations

For measurement, excite ion to a level where it is likely to spontaneously decay

Ion traps

Error rates for gates

Depends a lot on the implementation

For single-qubit gates, generally very good

$$0.001\% - 0.1\%$$

Same for measurement

For two-qubit gates

$$1\% - 20\%$$

Coherence times on the order of seconds and even minutes!

In 2017, a single-qubit quantum memory lasting 10 minutes

Ion traps

Coherence times go down for larger systems

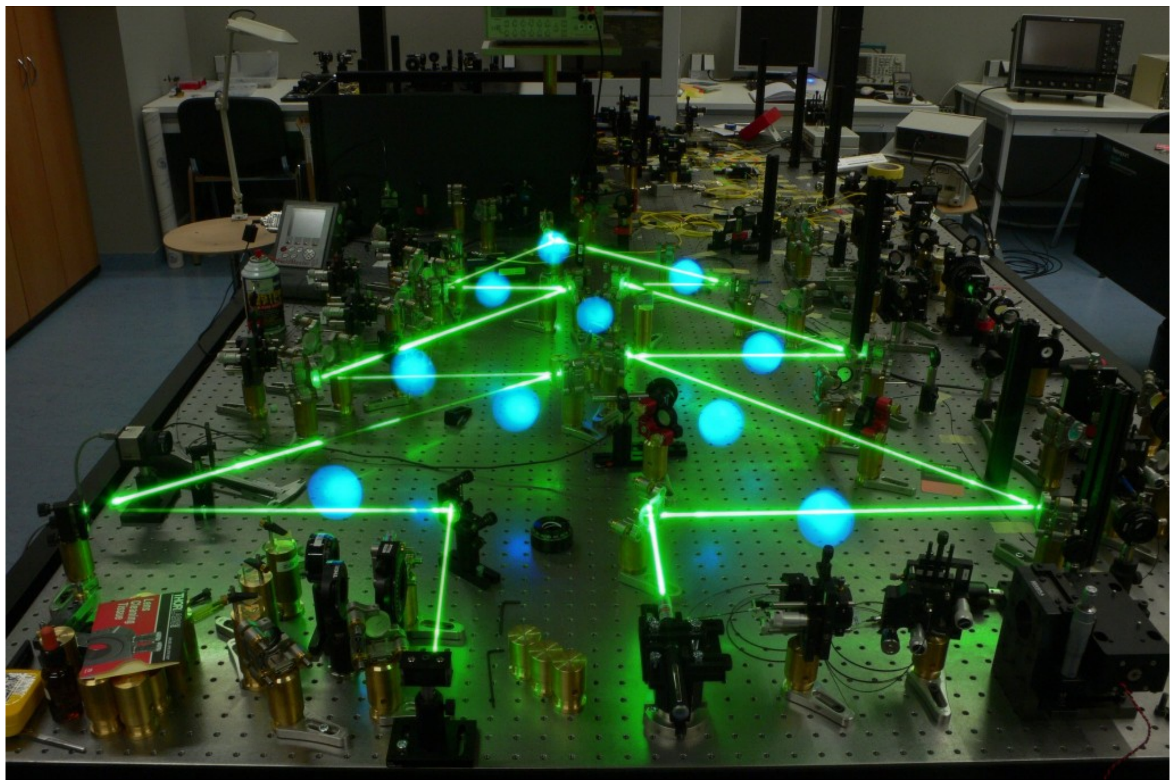
To date, largest entangled state on ions
14-qubits

Reported coherence time scaling for k ions $\frac{1}{k^2}$

Scalability is tricky, due to phonon interactions

Other implementations

Optical



Nuclear magnetic resonance

Hybrid (optical + ion traps)

NQIT Networked Quantum Information Technologies

References and resources

Energy levels and image on slide 11

https://en.wikipedia.org/wiki/Energy_level

https://physics.stackexchange.com/questions/188883/why-do-the-size-of-gaps-energy-between-different-energy-levels-of-mercury-hg-var

Superconducting qubits

https://arxiv.org/pdf/cond-mat/0411174.pdf

http://qulab.eng.yale.edu/documents/talks/Devoret-APS_Tutorial_090316s.pdf

https://qudev.phys.ethz.ch/content/courses/QSIT11/QSIT11_V05_slides.pdf

Source for images on slides 31, 33, 34

http://clelandlab.uchicago.edu/pdf/geller%20cleland%20qc%20architecture%20pra%202005.pdf

References and resources

Video lectures on superconducting qubits

https://www.youtube.com/watch?v=t5nxusm_Umk

https://www.youtube.com/watch?v=KOZCPl_DyDU

https://www.youtube.com/watch?v=c9WkyY5XSTw

https://www.youtube.com/watch?v=3c4xotTuIwE

https://www.youtube.com/watch?v=9ZJk2760KPE

Josephson junction and images on slides 21, 22, 23

https://en.wikipedia.org/wiki/Josephson_effect

http://hyperphysics.phy-astr.gsu.edu/hbase/Solids/Squid.html

Decoherence

https://en.wikipedia.org/wiki/Quantum_decoherence

https://plato.stanford.edu/entries/qm-decoherence/

https://www.scottaaronson.com/democritus/lec11.html

References and resources

Gate errors and coherence times for IBM

https://quantumexperience.ng.bluemix.net/qx/devices

Gate errors and coherence times for Rigetti

http://docs.rigetti.com/en/stable/qpu.html

Gate errors and coherence times for Google

https://www.youtube.com/watch?v=SCTGI5aGqJU

https://www.wired.co.uk/article/googles-head-of-quantum-computing

D-Wave coherence times

https://arxiv.org/pdf/1503.08955.pdf

https://arxiv.org/pdf/1512.07617.pdf

References and resources

Ion trap quantum computing

https://qudev.phys.ethz.ch/phys4/studentspresentations/iontraps/CiracZoller1995.pdf

https://en.wikipedia.org/wiki/Trapped_ion_quantum_computer

https://www.icfo.es/images/publications/Proc.06-002.pdf

Section 7.6 in Nielsen and Chuang

More on ion traps and images from slides 42, 43

https://physics.aps.org/articles/v7/119

https://quantumoptics.at/en/news/72-scalable-multiparticle-entanglement-of-trapped-ions.html

Hyperfine energy levels and image on slide 45

https://en.wikipedia.org/wiki/Hyperfine_structure

https://en.wikipedia.org/wiki/Hydrogen_line

References and resources

Position quantisation (particle in a box)
and image on slide 46
http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/pbox.html
https://en.wikipedia.org/wiki/Particle_in_a_box

Ion traps gate errors and coherence times
https://physics.aps.org/featured-article-pdf/10.1103/PhysRevLett.113.220501
https://arxiv.org/pdf/1512.04600.pdf
https://arxiv.org/pdf/1701.04195.pdf

14-qubit entangled state on ions
https://arxiv.org/pdf/1009.6126.pdf

References and resources

Optical quantum computing

https://arxiv.org/pdf/quant-ph/0512071.pdf

https://en.wikipedia.org/wiki/Linear_optical_quantum_computing

NMR quantum computing

https://en.wikipedia.org/wiki/Nuclear_magnetic_resonance_quantum_computer

http://science.sciencemag.org/content/277/5332/1688

NQIT

https://nqit.ox.ac.uk/

https://nqit.ox.ac.uk/content/ion-traps

https://nqit.ox.ac.uk/content/atom-photon-interfaces

Section 7 of Nielsen and Chuang

References and resources

Images on slide 53

http://quantum.opticsolomouc.org/archives/876

https://www.msconnection.org/Blog/June-2015/(Not)-The-Sound-Of-Music