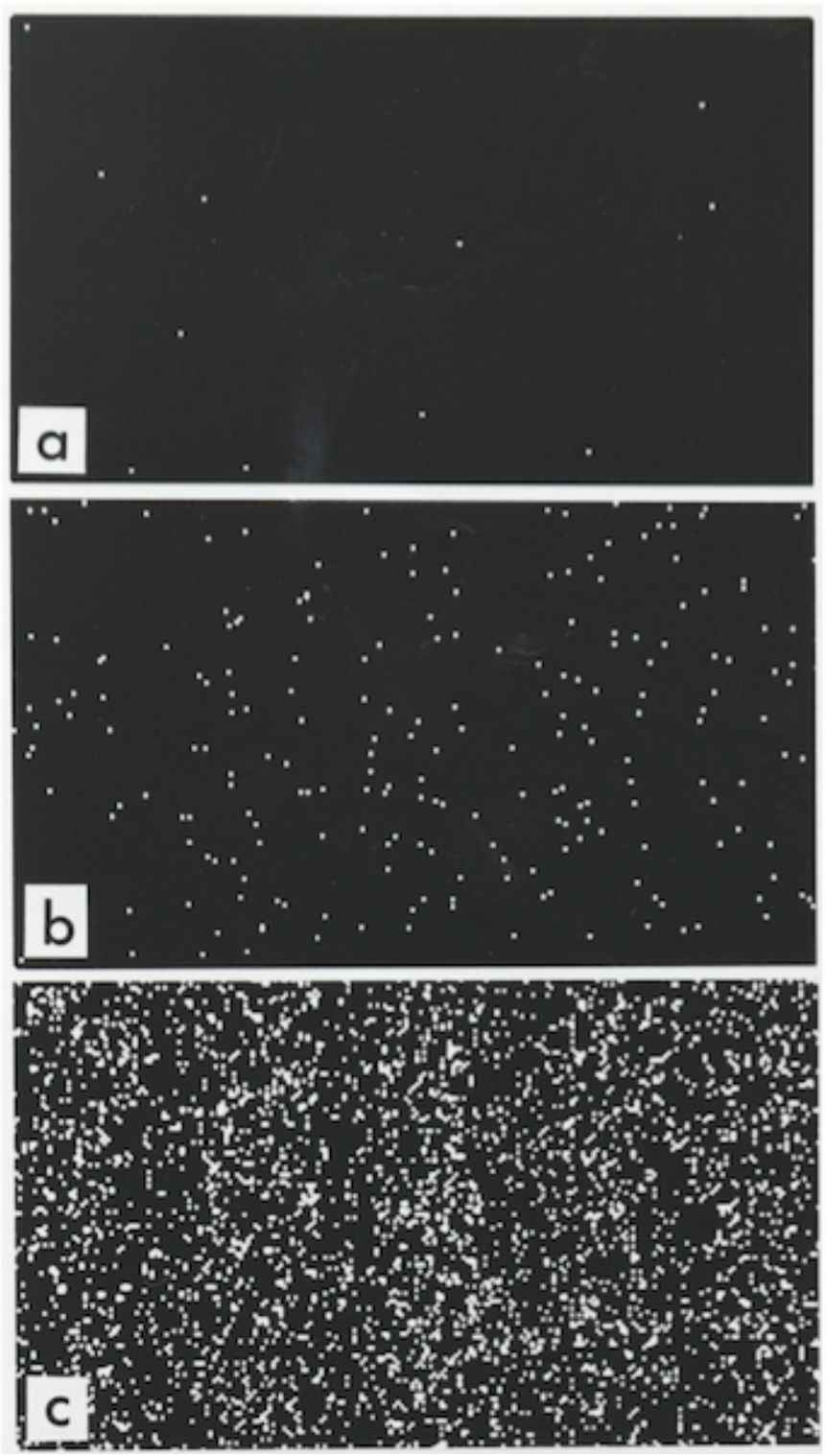
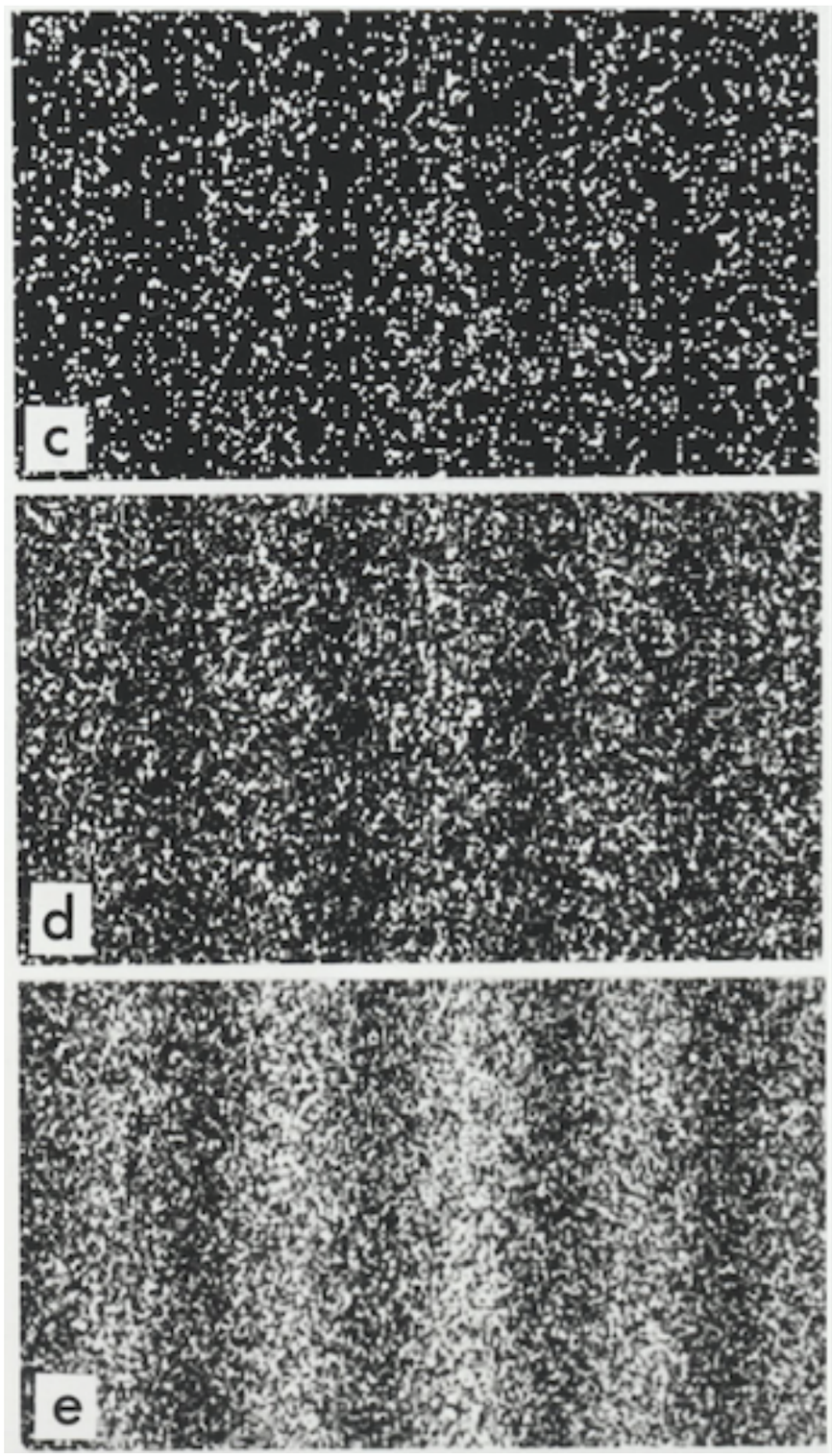
Quantum Computation & Cryptography
Day 1

What's quantum about quantum computing?

Andru Gheorghiu

The answer is **interference**





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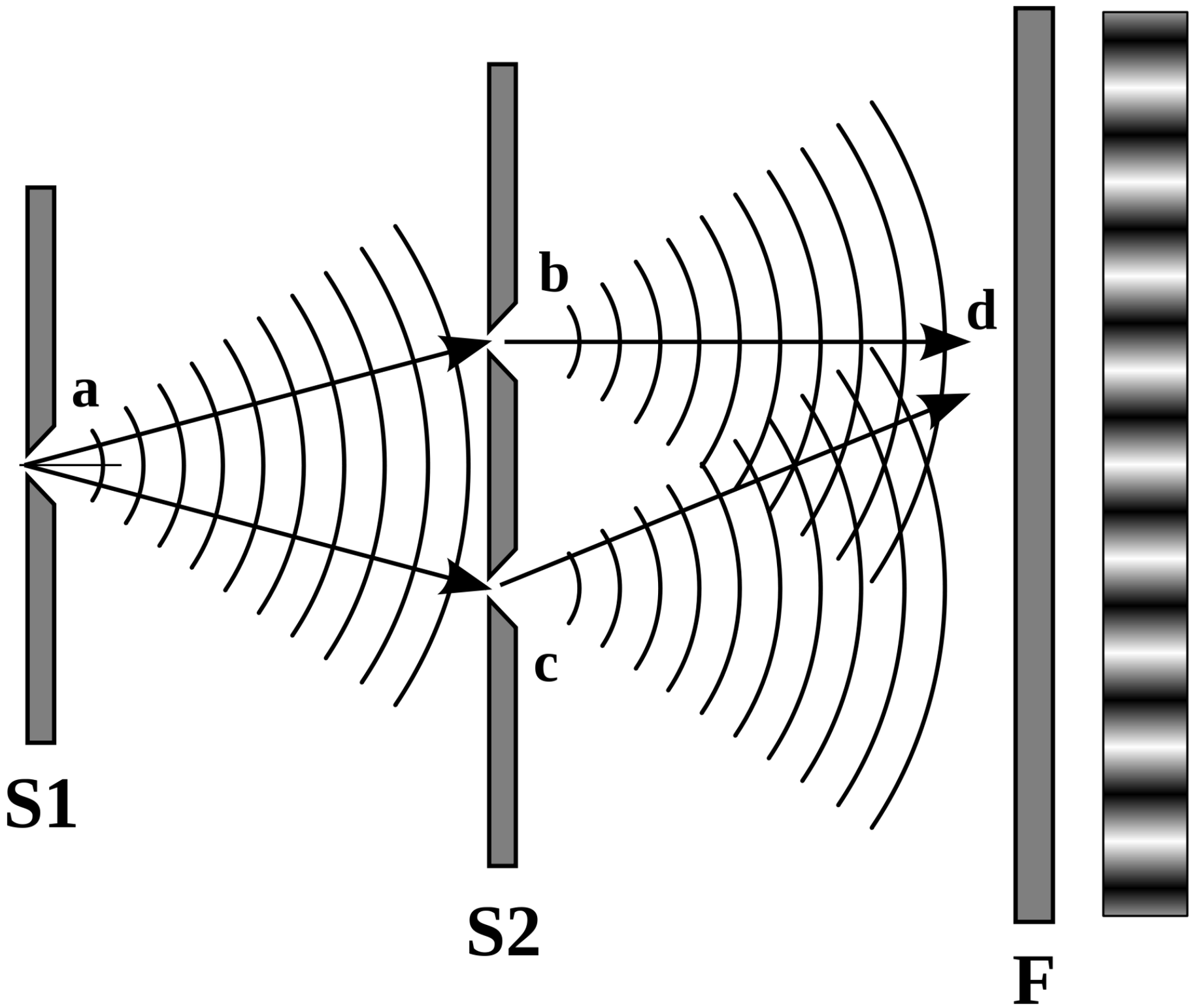


"*We choose to examine a phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality it contains the only mystery.*"

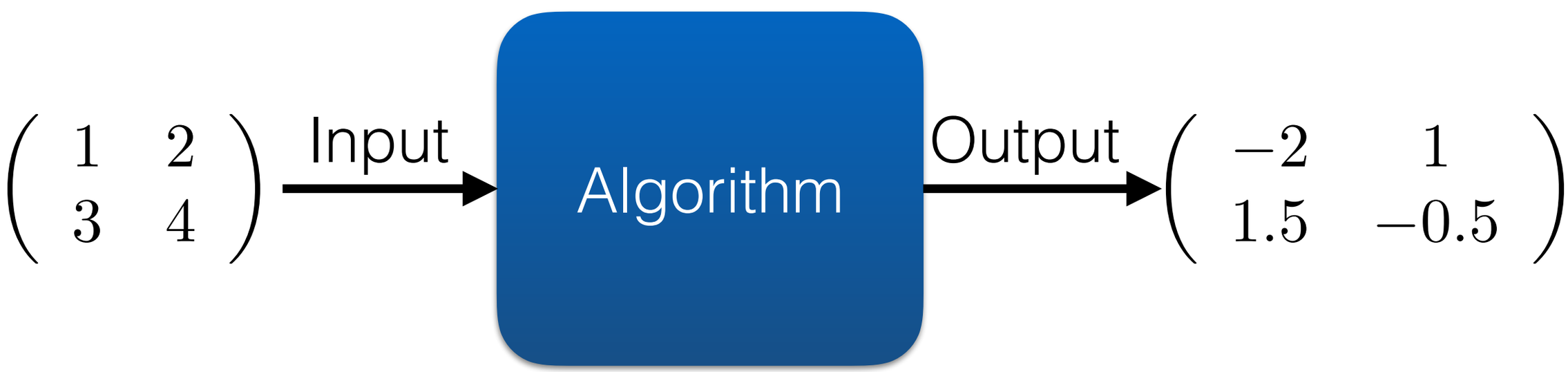
Computation

Computation

Sorting

Computation

Matrix inversion



Computation

Primality testing

Efficient computation

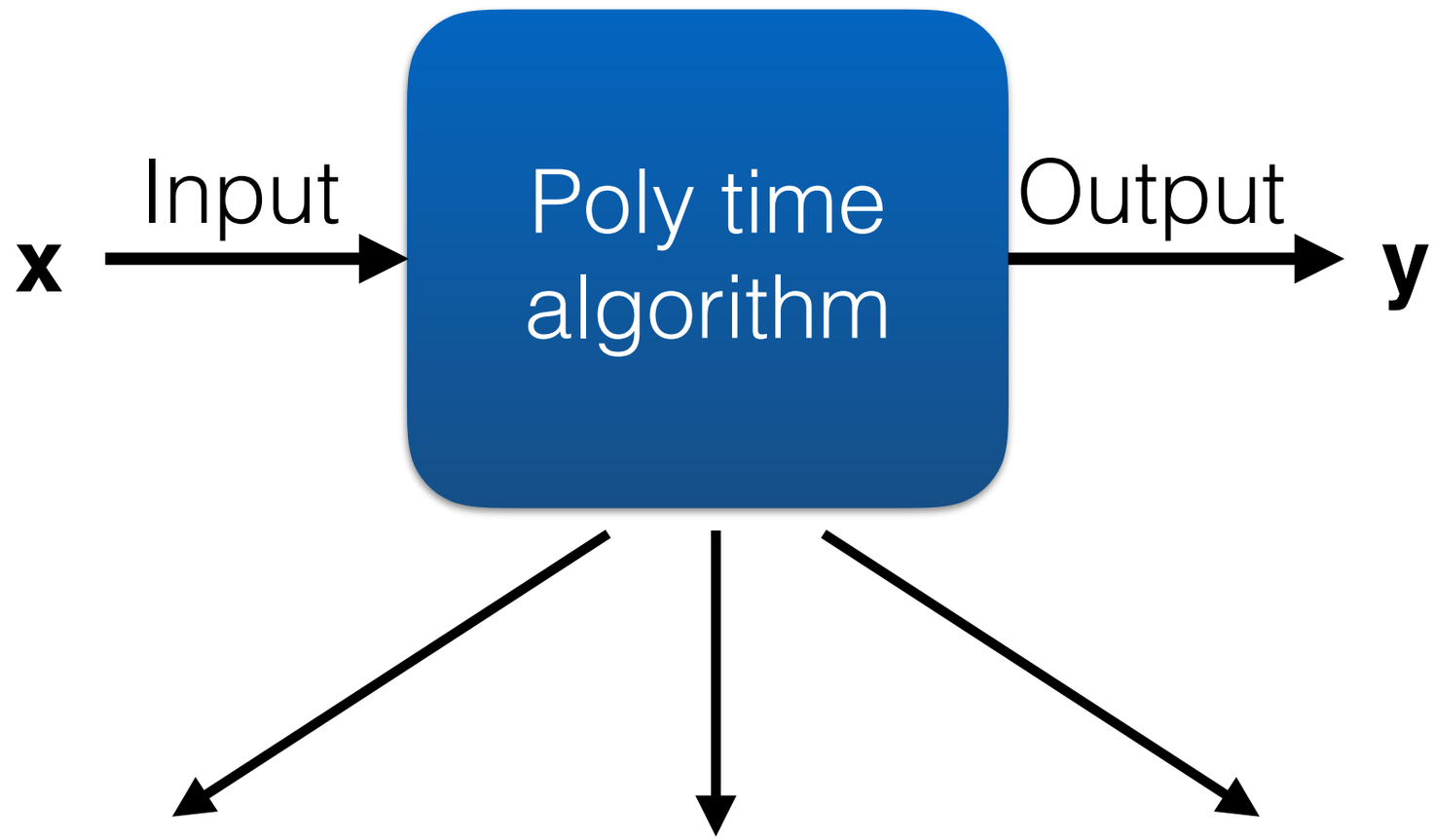
What does efficient algorithm mean?

Number of instructions is a *polynomial* in length of input

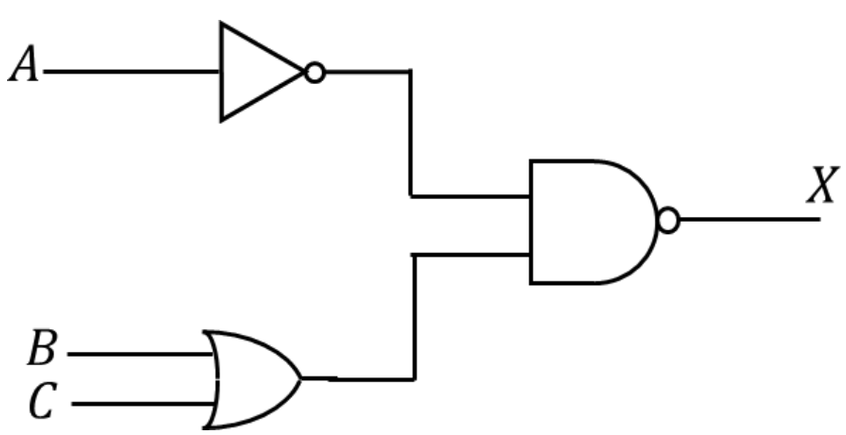
There is a constant d > 0 such that for all **x** running time is $O(|x|^d)$

There are caveats to this definition!

Types of computation

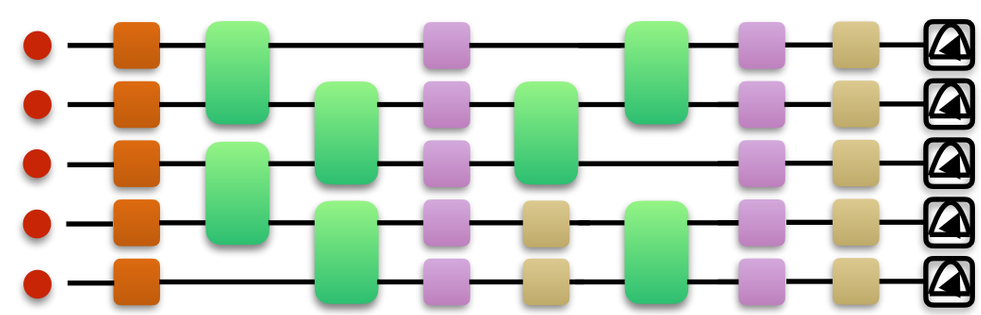


Deterministic



Probabilistic

Quantum



Deterministic computation

Write some code…

```
while (!toVisit.isEmpty()) {
    // remove queue head
    Pair<String, Integer> currentPair = toVisit.remove();
    String currentNode = currentPair.getFst();
    int distance = currentPair.getSnd();

    // reached destination?
    if (currentNode.equals(dest))
        return distance;

    // already visited this node, skip
    if (visited.contains(currentNode)) continue;

    // add neighbors to queue, to be visited later on
    for (String neighbor : neighbors.get(currentNode)) {
        // reached destination?
        if (neighbor.equals(dest))
            return distance + 1;
        toVisit.add(new Pair<String, Integer>(neighbor, distance + 1));
    }
    visited.add(currentNode);
}
```

Deterministic computation

Compile the code…

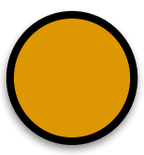
```
00000000    push    ebp
00000001    mov     ebp, esp
00000003    movzx   ecx, [ebp+arg_0]
00000007    pop     ebp
00000008    movzx   dx, cl
0000000C    lea     eax, [edx+edx]
0000000F    add     eax, edx
00000011    shl     eax, 2
00000014    add     eax, edx
00000016    shr     eax, 8
00000019    sub     cl, al
0000001B    shr     cl, 1
0000001D    add     al, cl
0000001F    shr     al, 5
00000022    movzx   eax, al
00000025    retn
```

Deterministic computation

Run the code…

Deterministic computation

x



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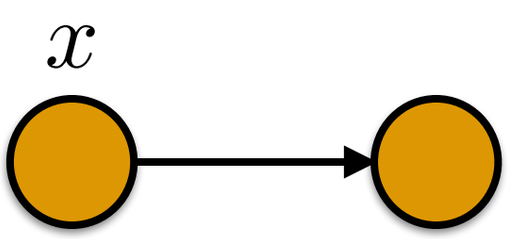

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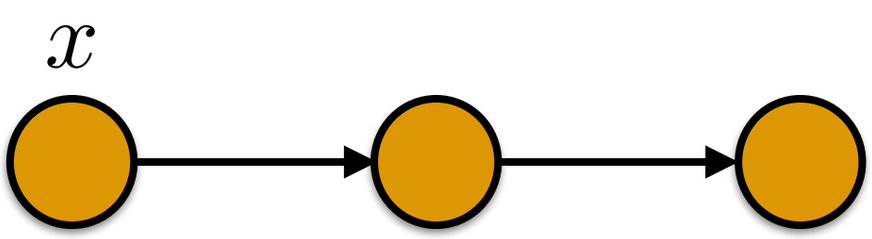

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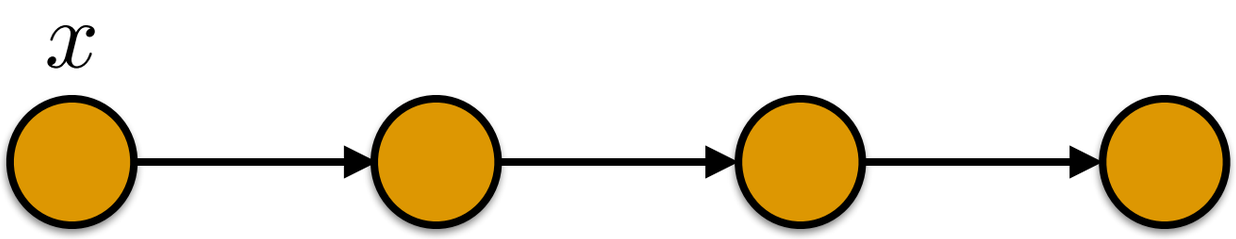

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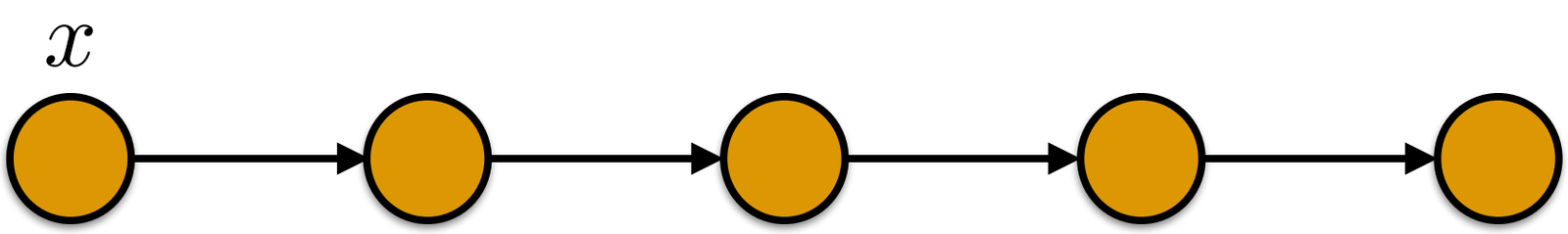

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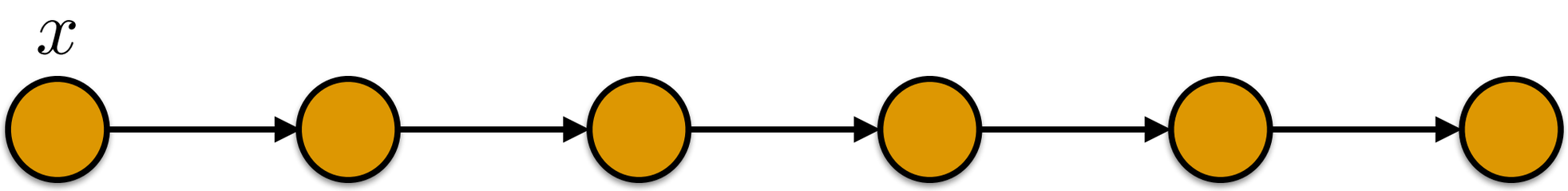

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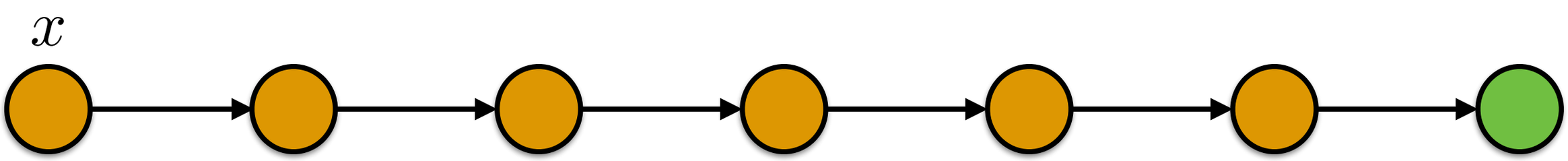

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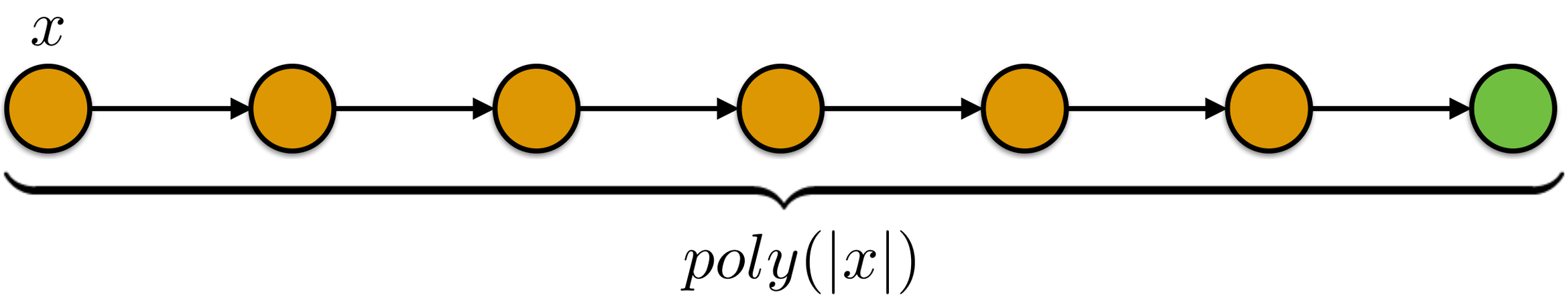

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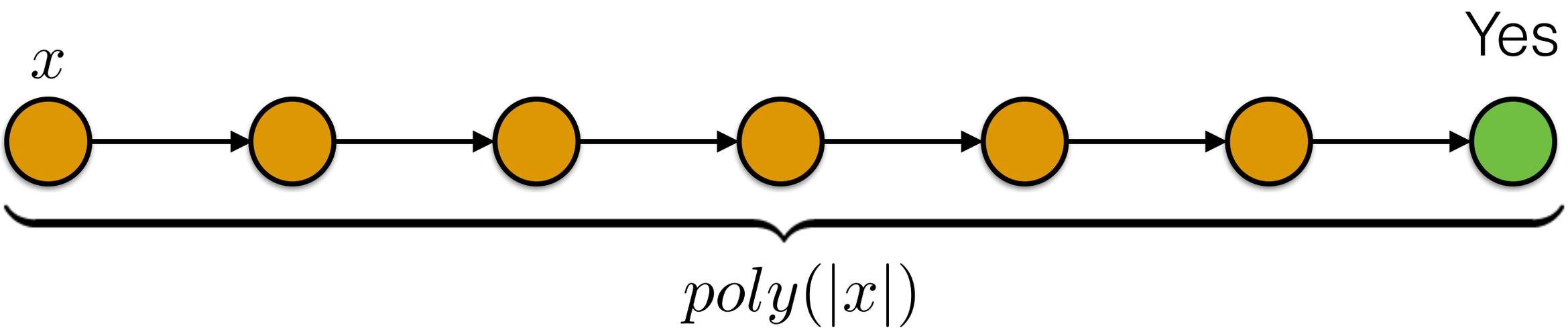

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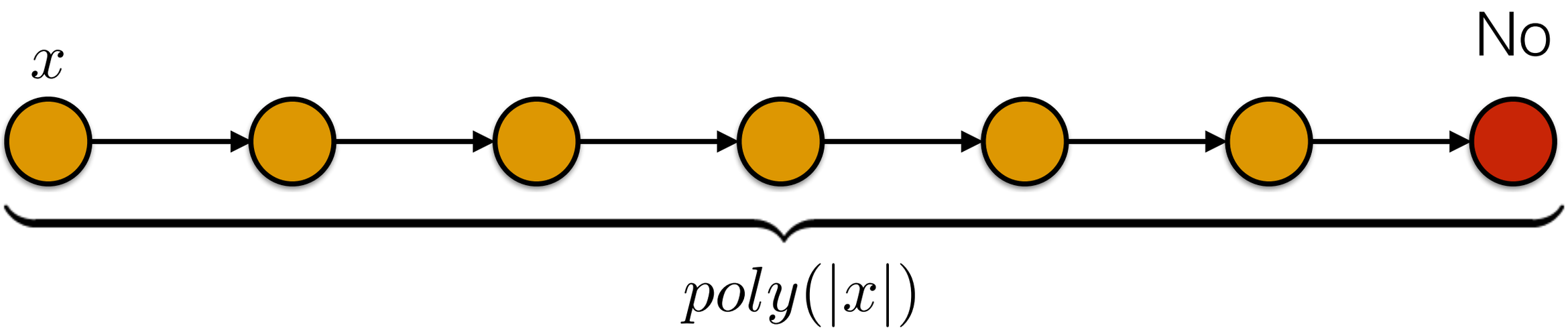

Probabilistic computation

Let’s add some randomness

Probabilistic computation

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As if computer flips a coin every now and then

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As if computer flips a coin every now and then

We allow the algorithm to err some of the time

Probabilistic computation

Let's add some randomness

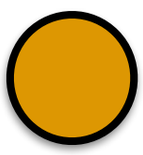
As if computer flips a coin every now and then

We allow the algorithm to err some of the time

Should obtain correct answer <u>with high probability</u>

Probabilistic computation

x



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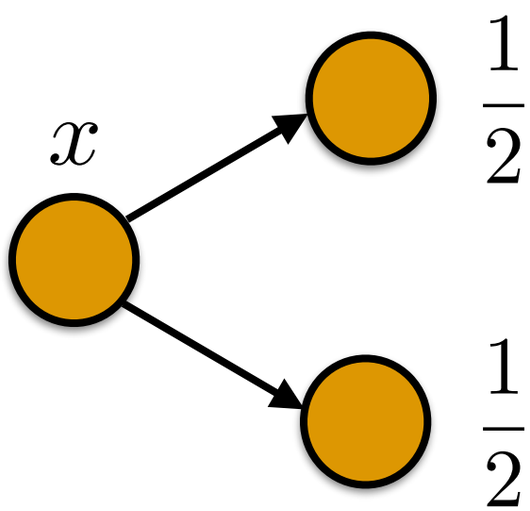

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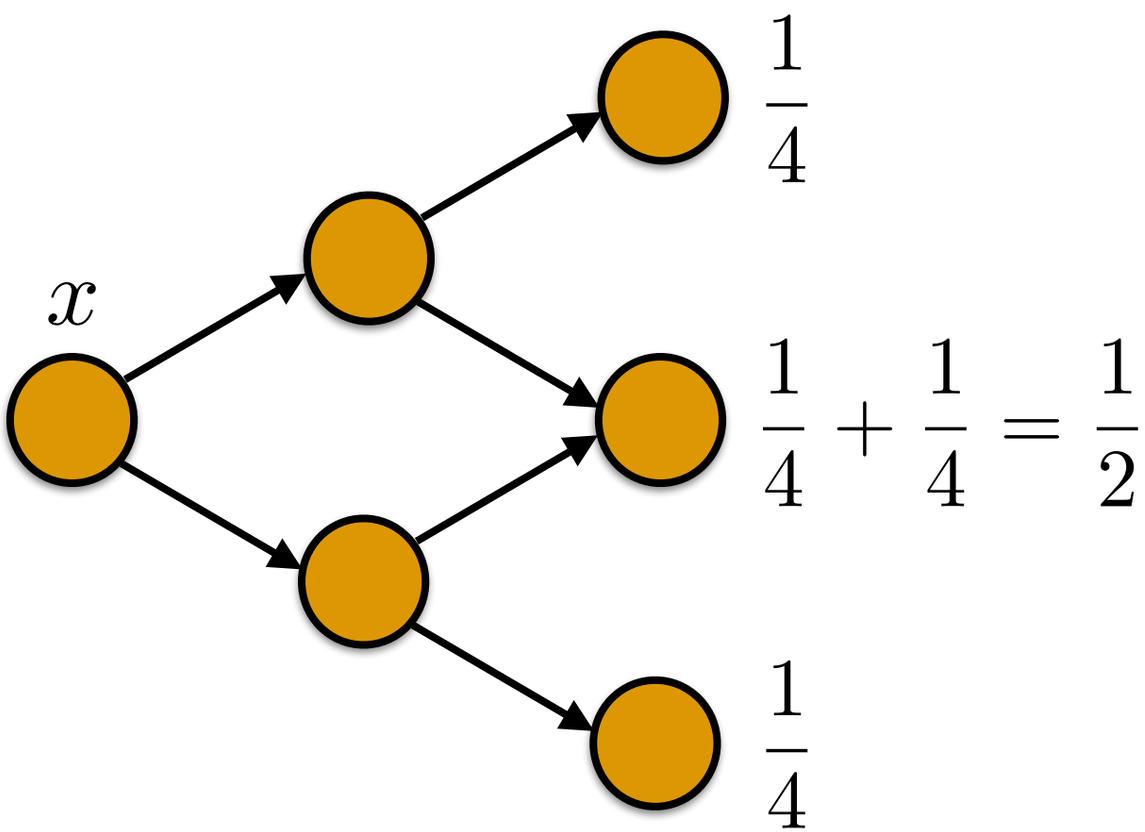

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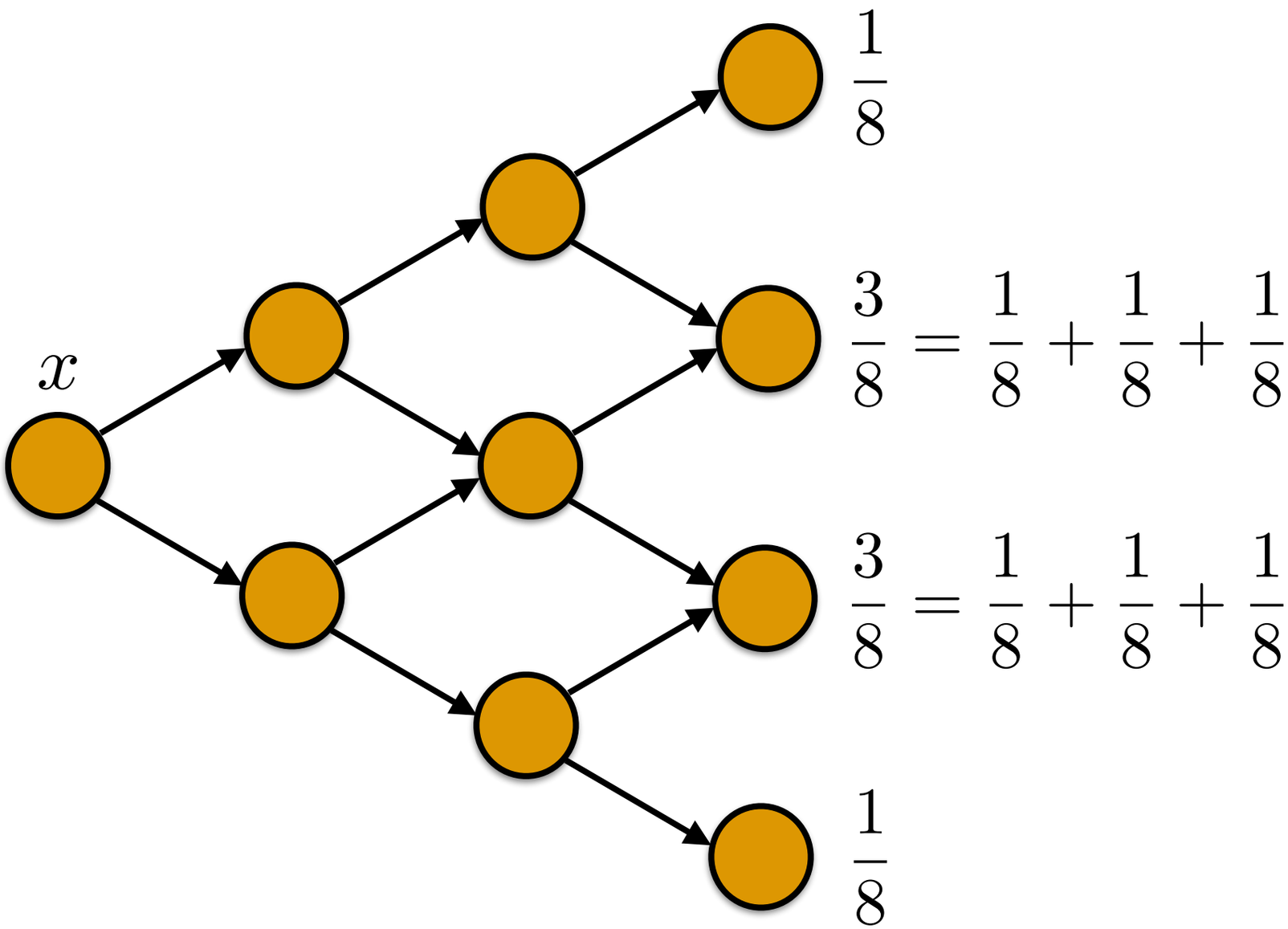

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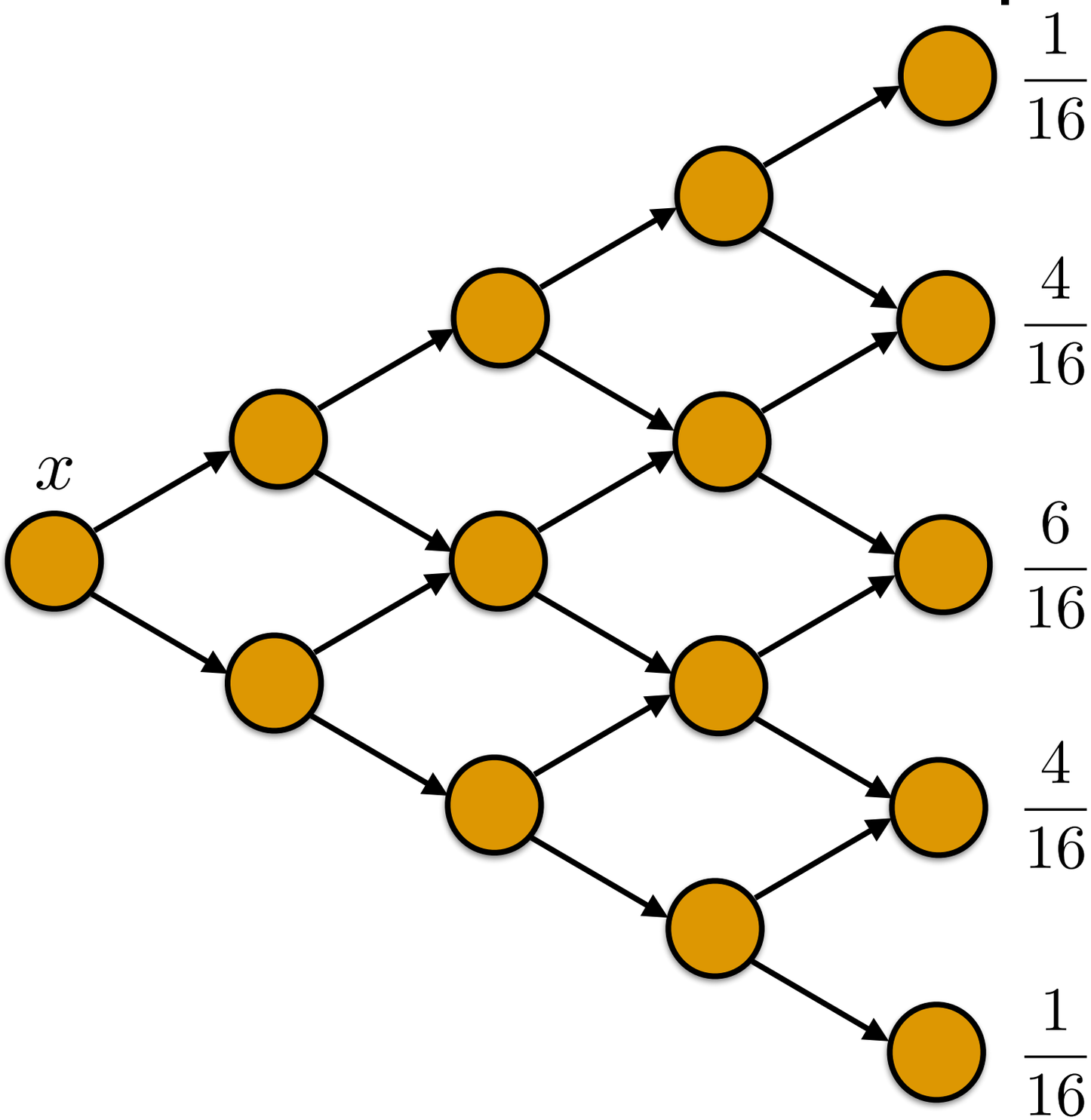

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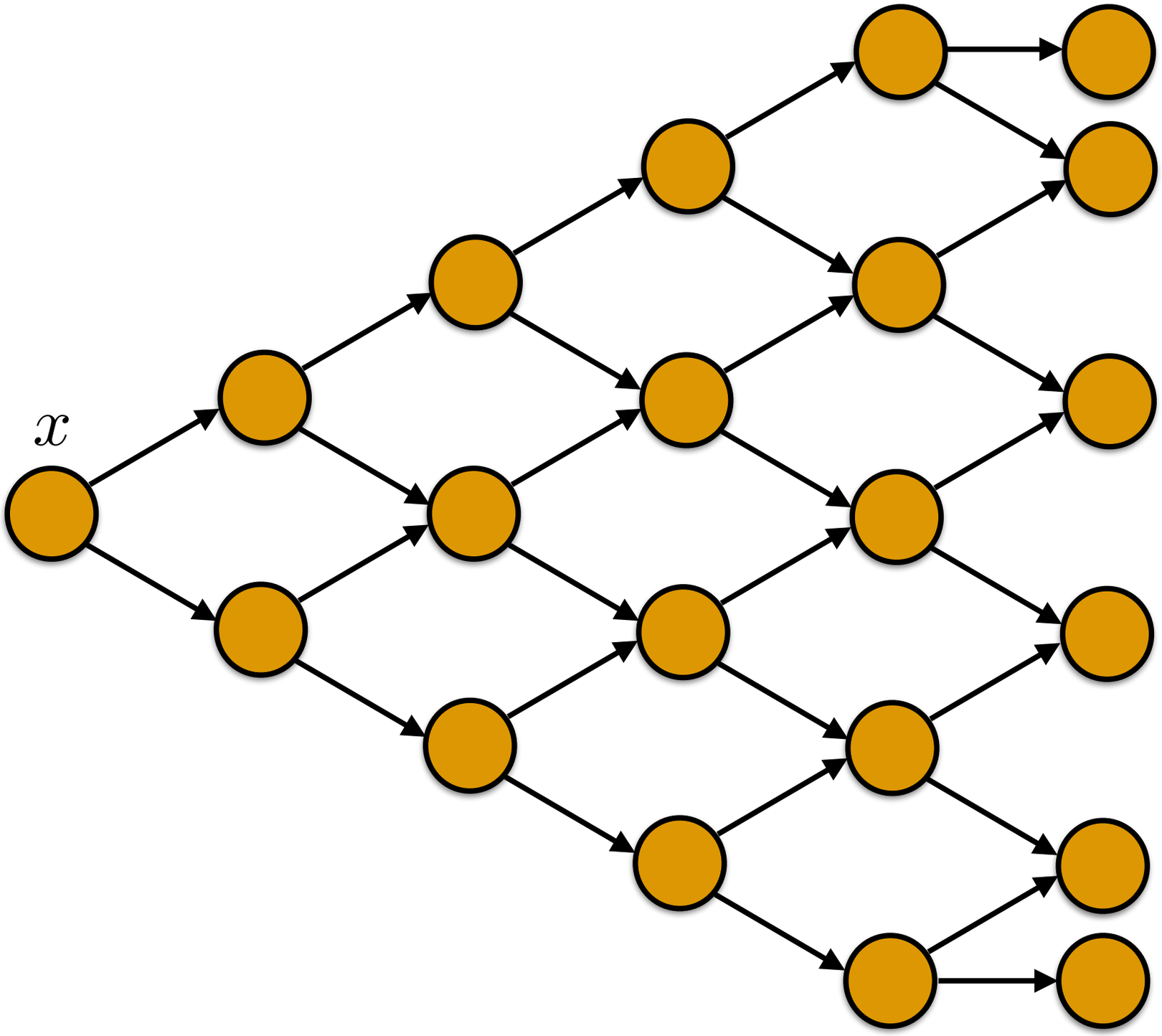

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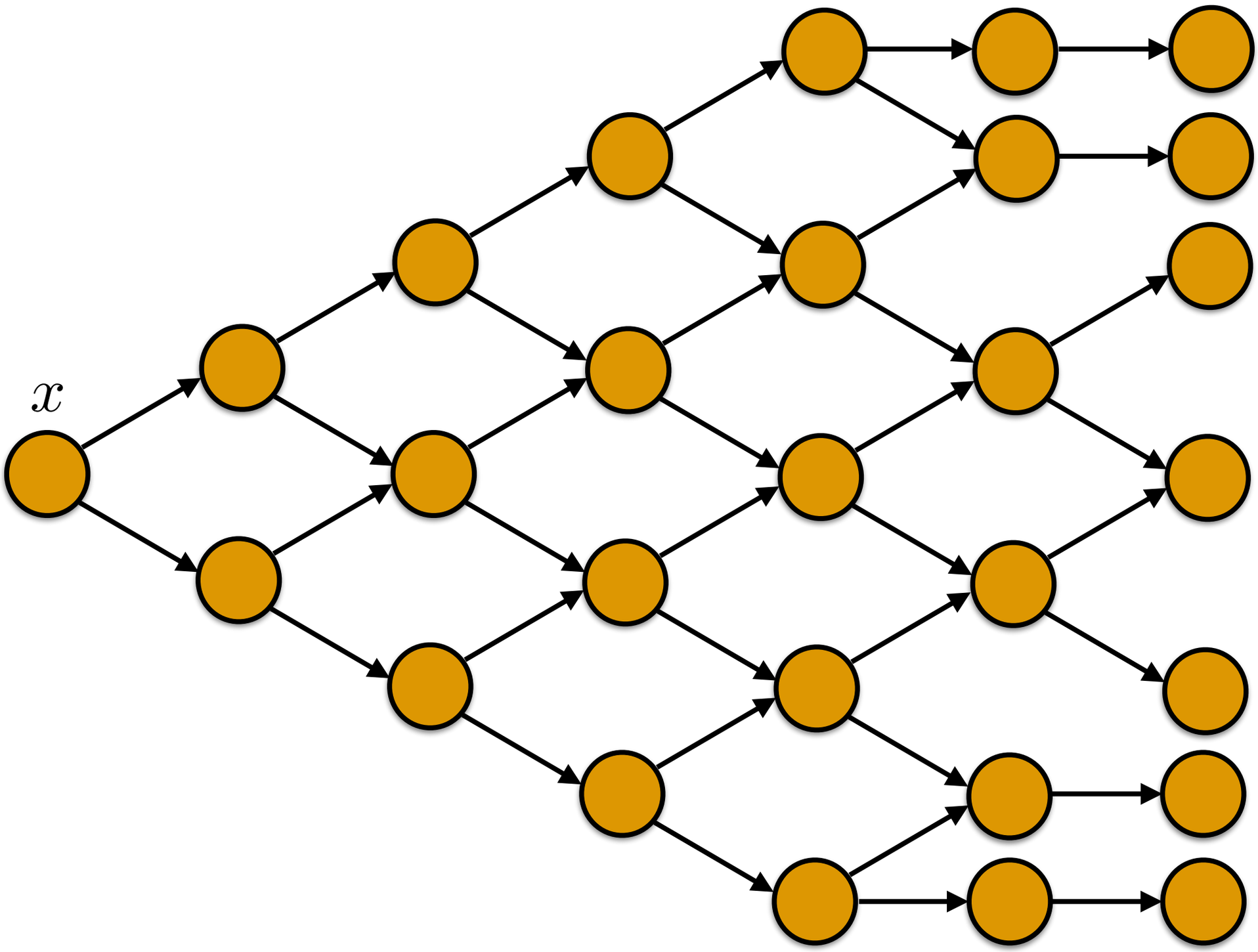

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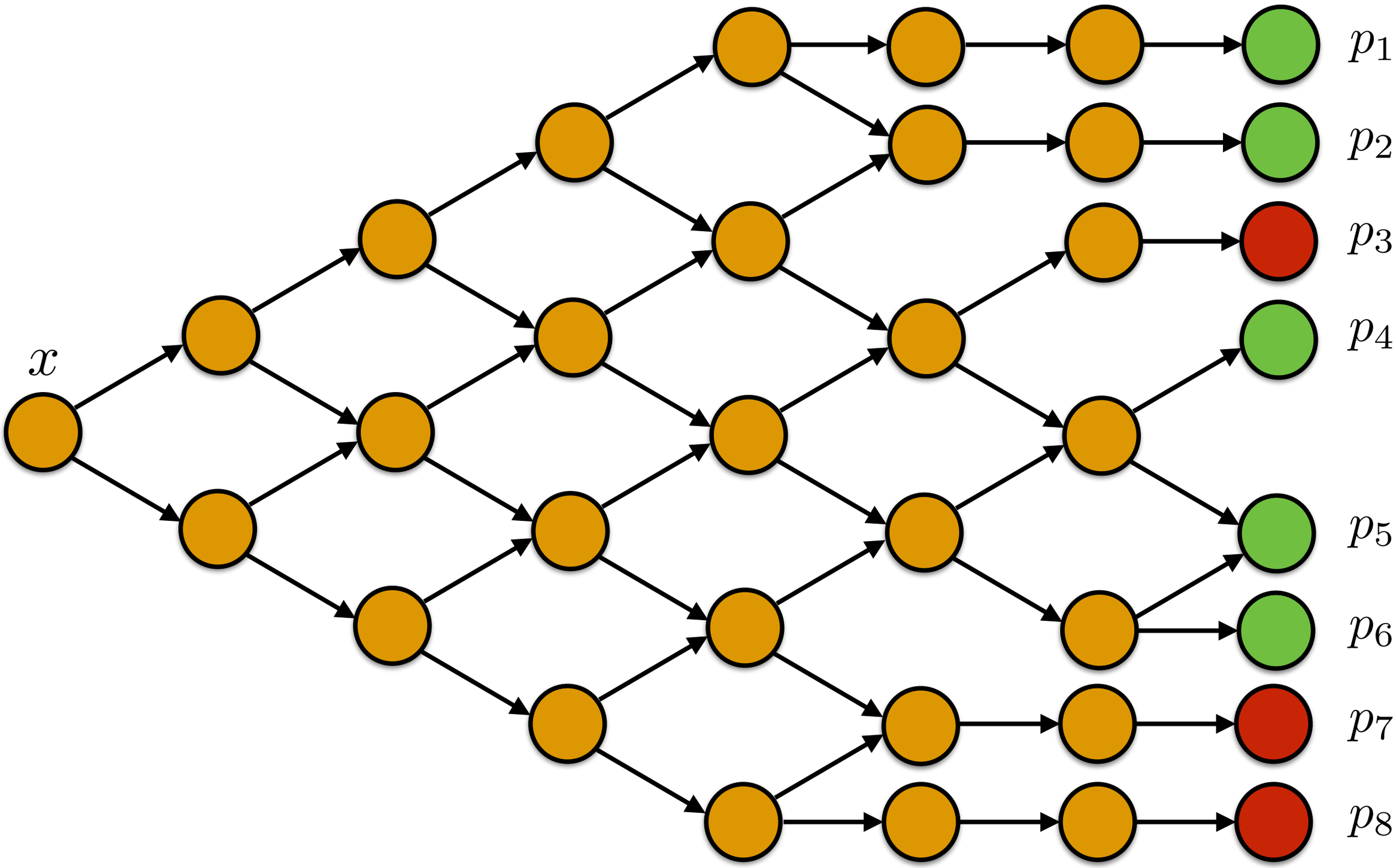

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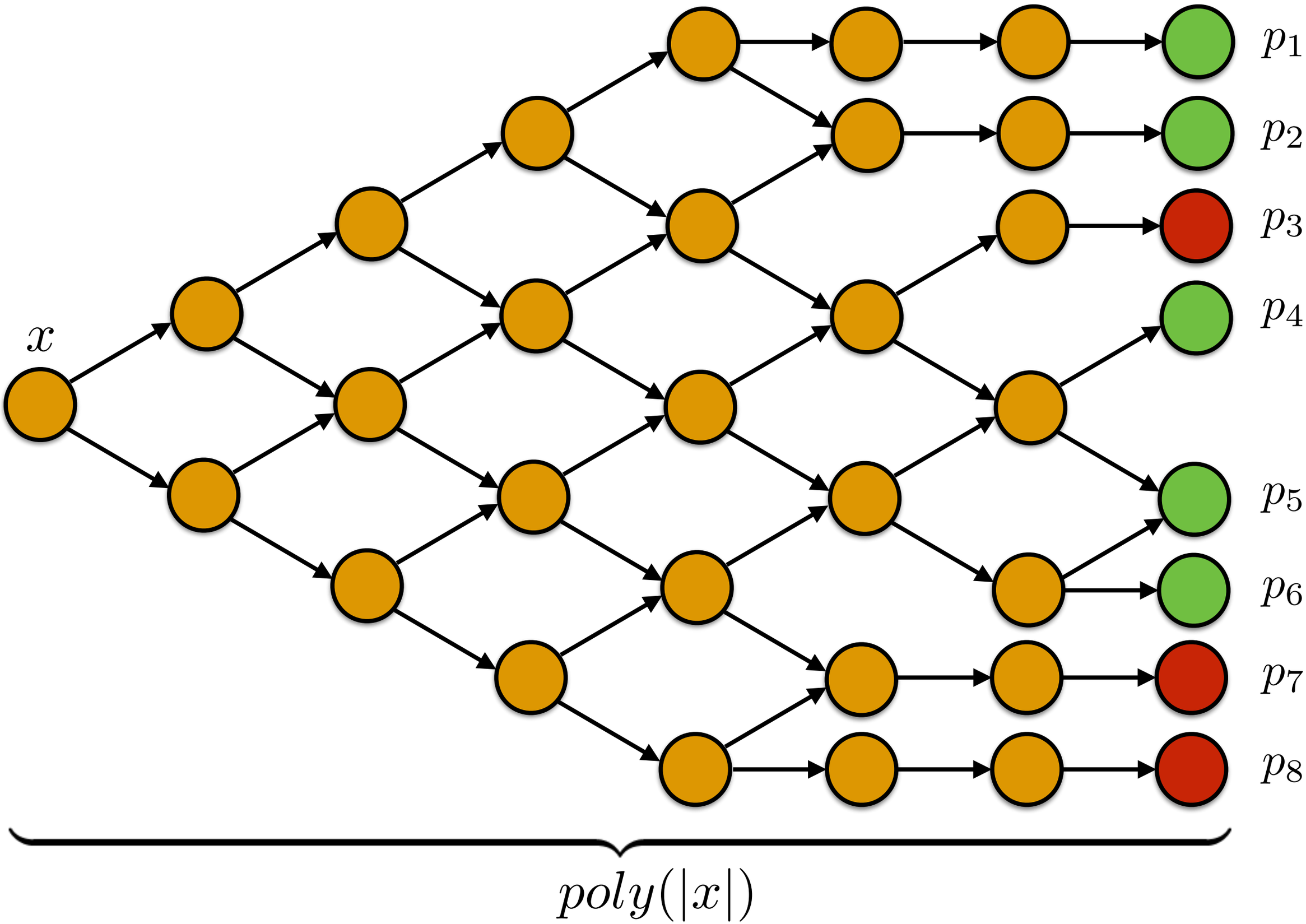

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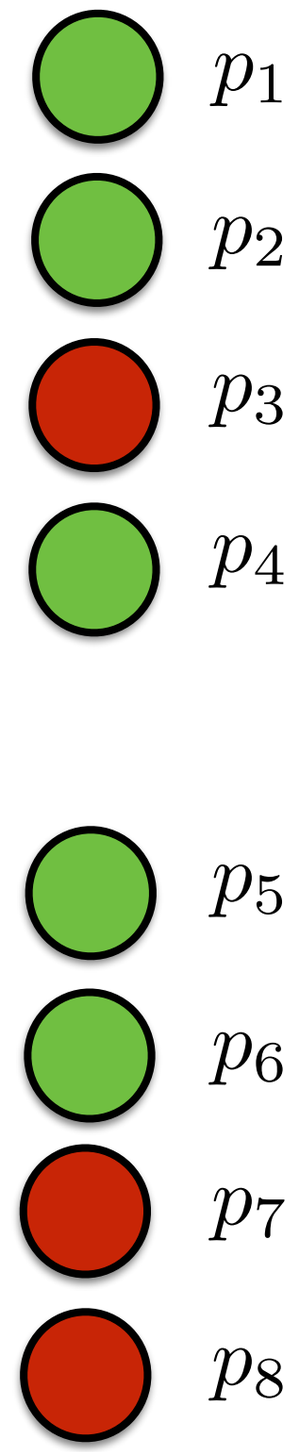

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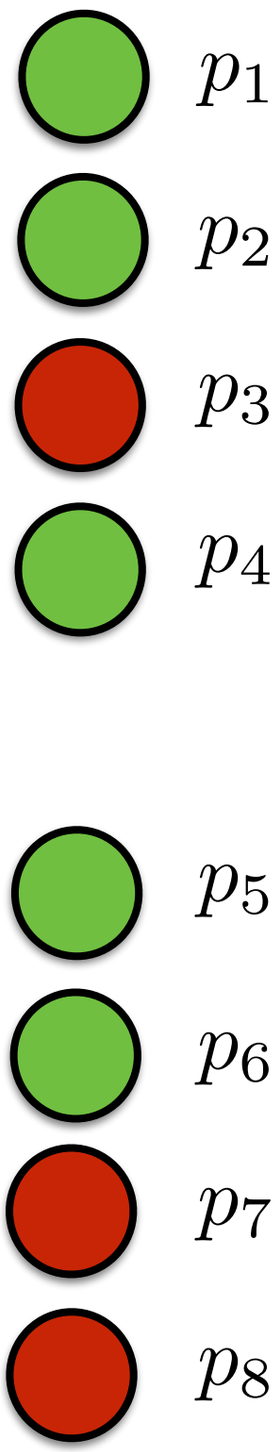
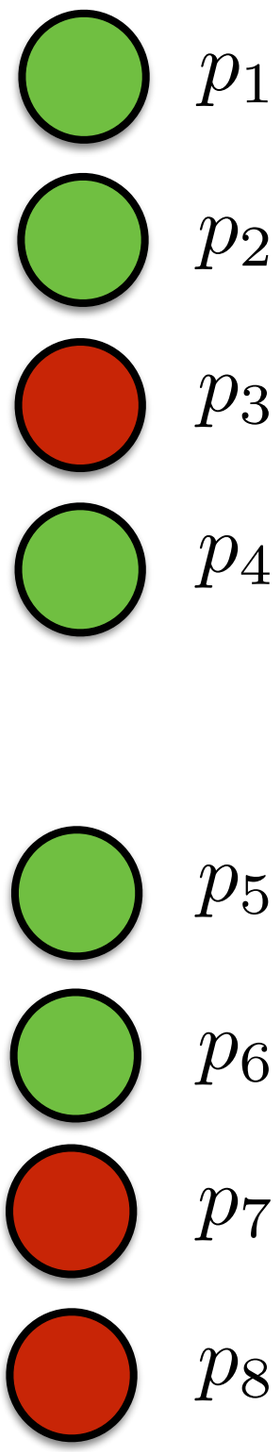


$$Pr(YES) = p_1 + p_2 + p_4 + p_5 + p_6$$

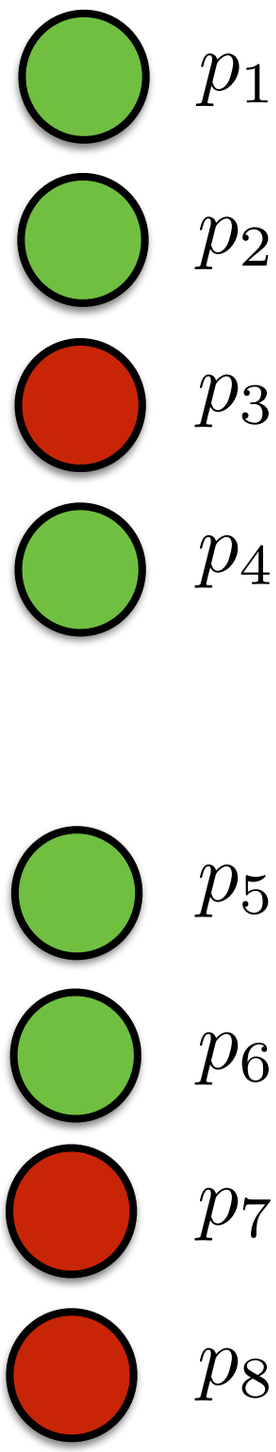
Probabilistic computation



$$Pr(YES) = p_1 + p_2 + p_4 + p_5 + p_6$$

$$Pr(NO) = p_3 + p_7 + p_8$$

Probabilistic computation

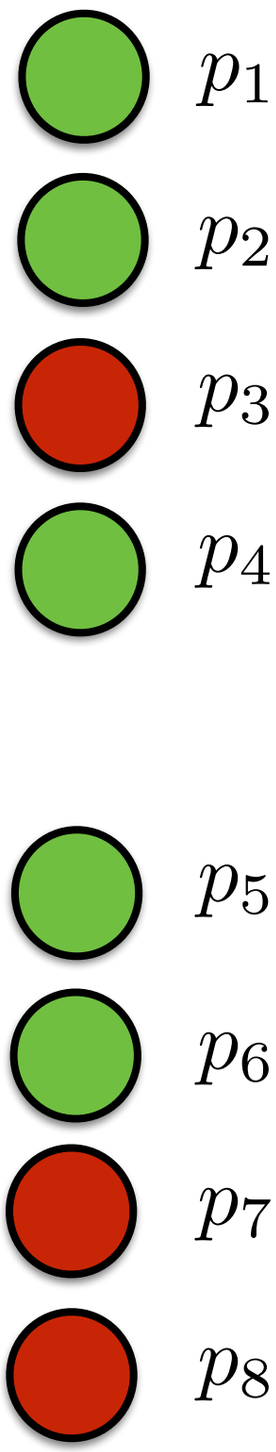


$$Pr(YES) = p_1 + p_2 + p_4 + p_5 + p_6$$

$$Pr(NO) = p_3 + p_7 + p_8$$

$$Pr(YES) + Pr(NO) = 1$$

Probabilistic computation



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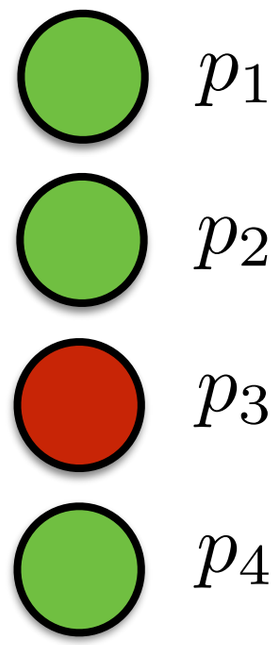
$$Pr(YES) + Pr(NO) = 1$$

When answer is YES, Pr(YES) > 2/3

Probabilistic computation



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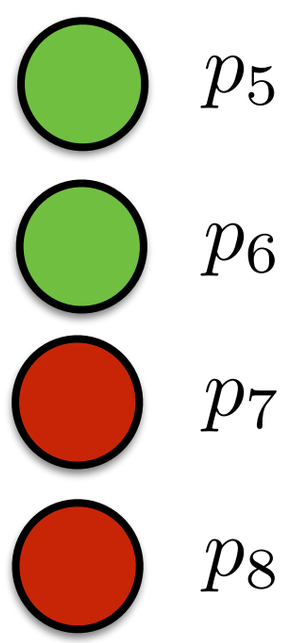
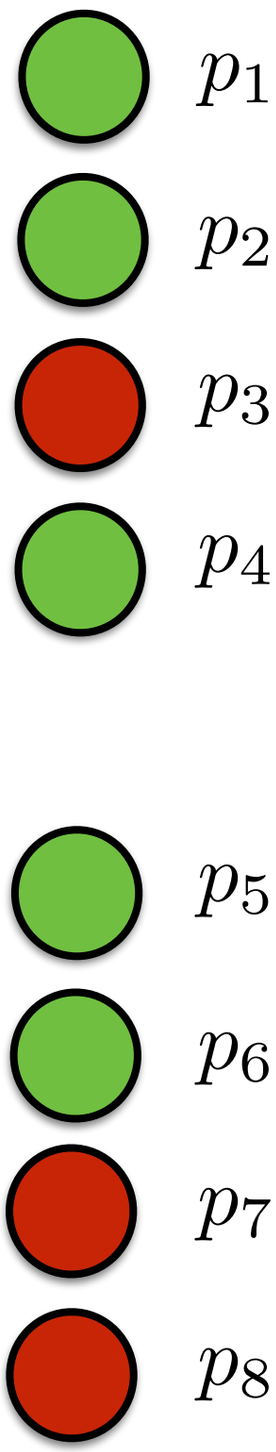


When answer is YES, Pr(YES) > 2/3

When answer is NO, Pr(NO) > 2/3

Probabilistic computation



$$Pr(YES) = p_1 + p_2 + p_4 + p_5 + p_6$$

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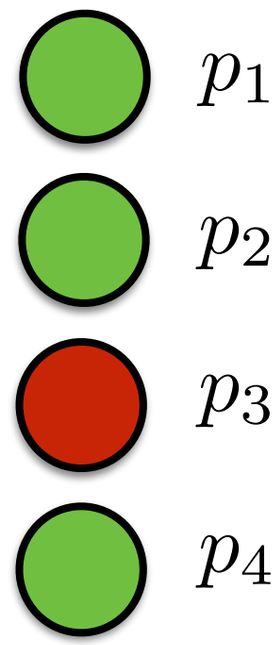
When answer is YES, Pr(YES) > 2/3

When answer is NO, Pr(YES) < 1/3

Probabilistic computation



$$Pr(YES) = p_1 + p_2 + p_4 + p_5 + p_6$$

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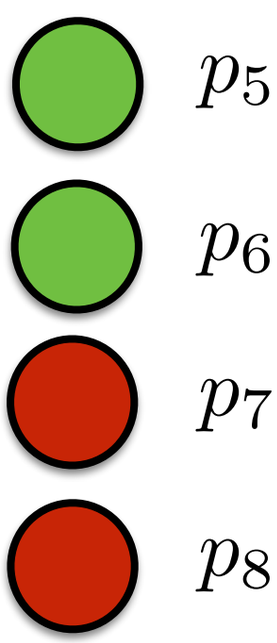
$$Pr(YES) + Pr(NO) = 1$$



When answer is YES, Pr(YES) > 2/3

When answer is NO, Pr(YES) < 1/3

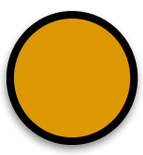
This defines a probabilistic poly time computation!

Quantum computation

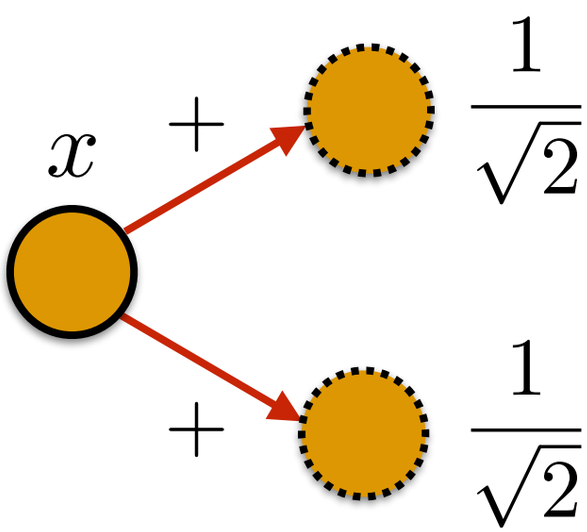
Quantum computers use “quantum probabilities”
(i.e. amplitudes)

Quantum computation

x



Quantum computation



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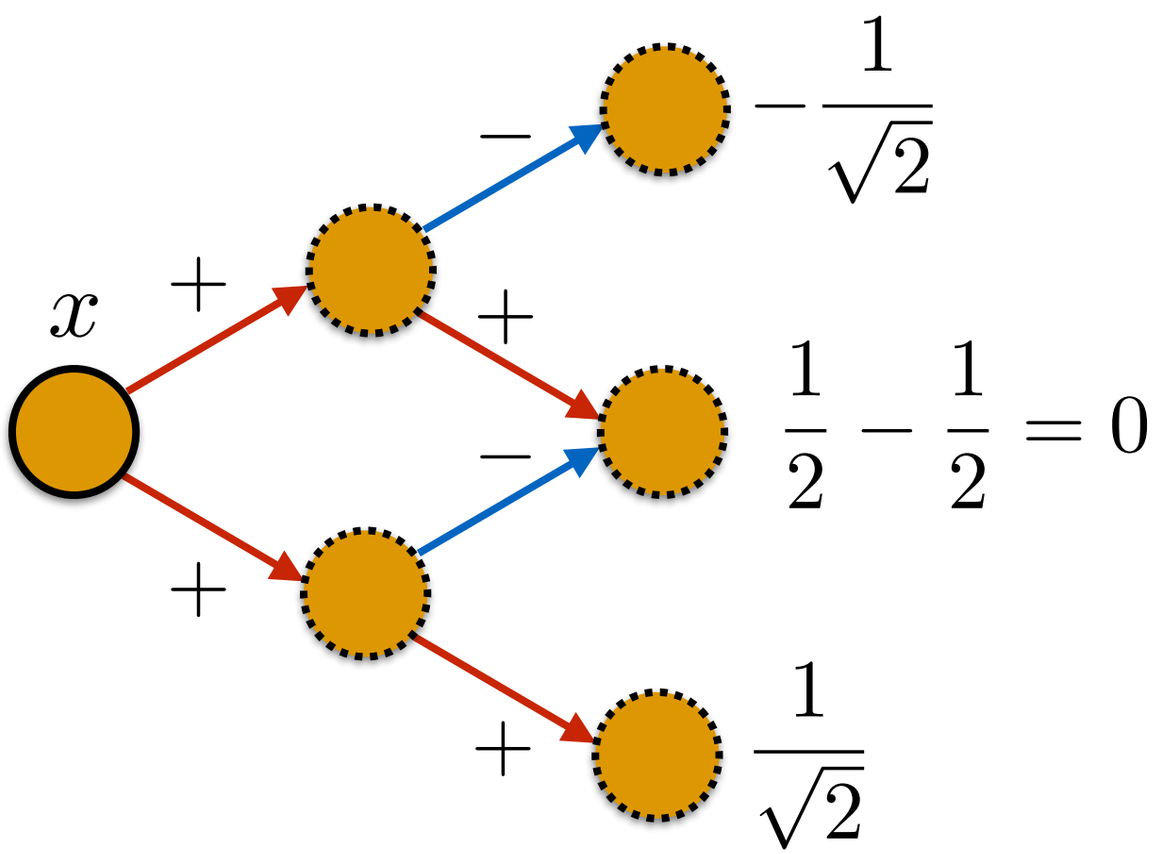

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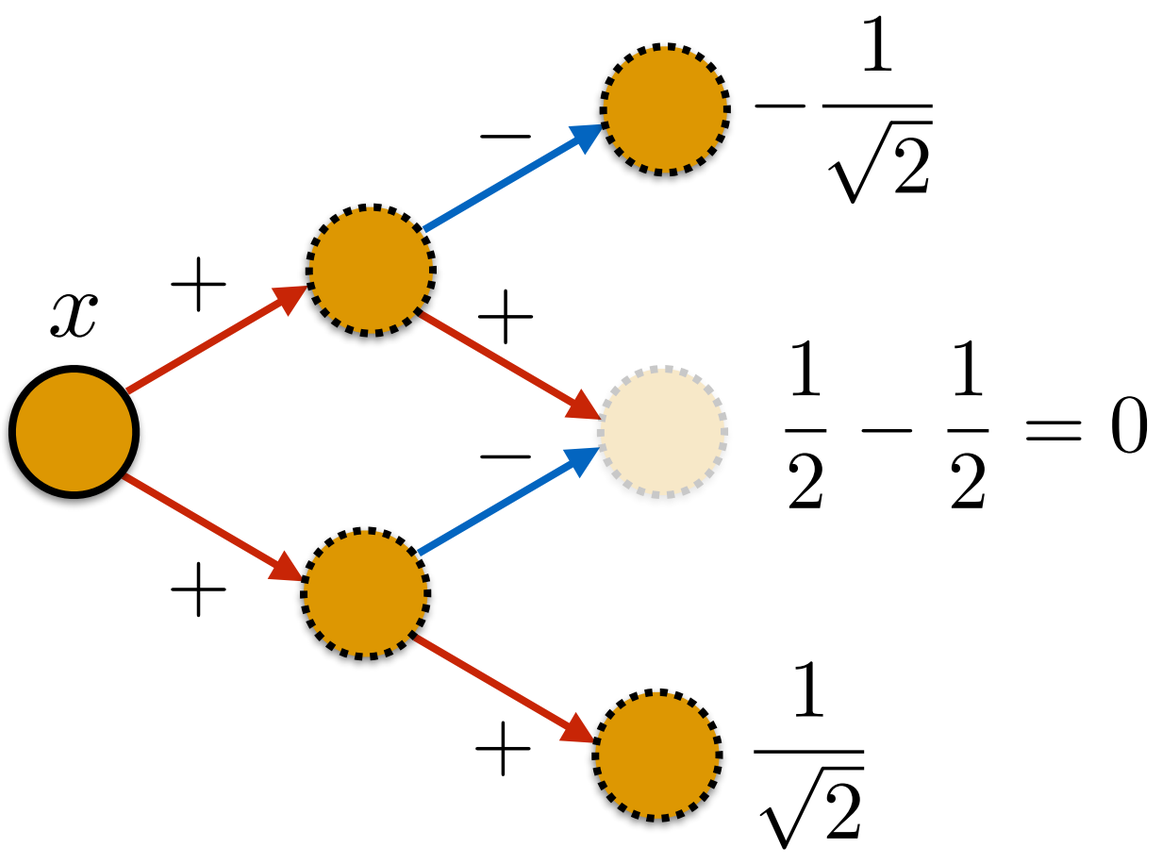

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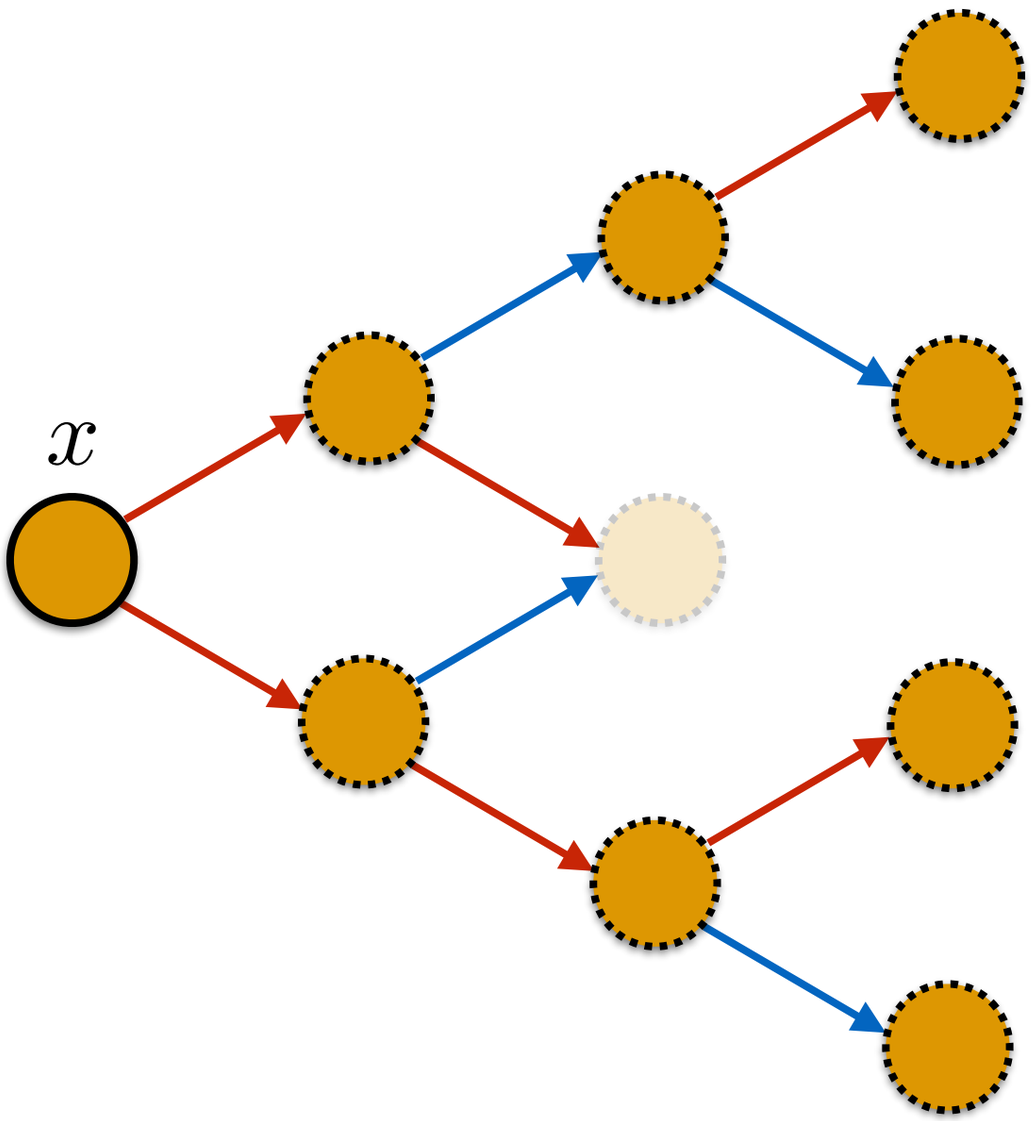

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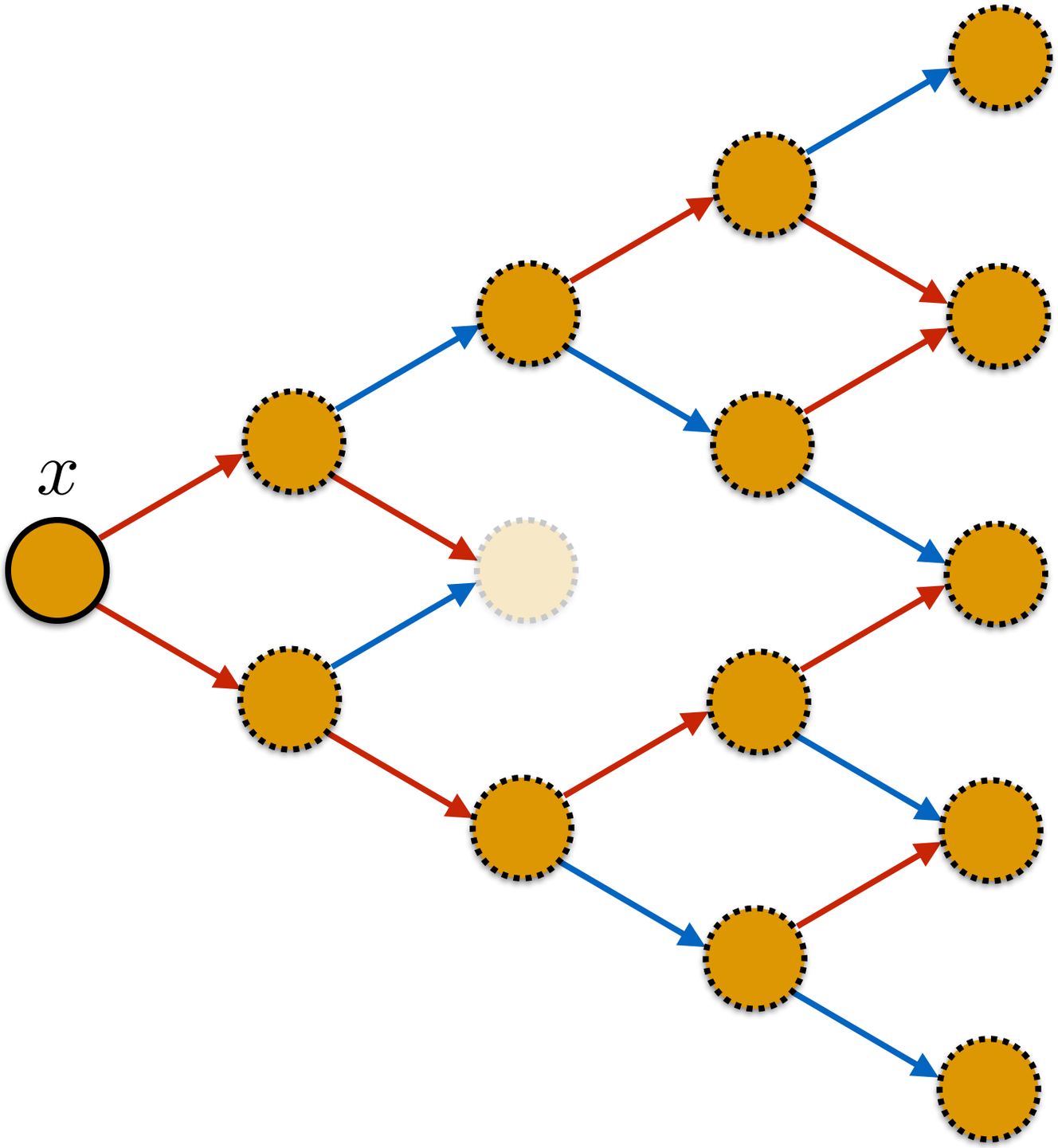

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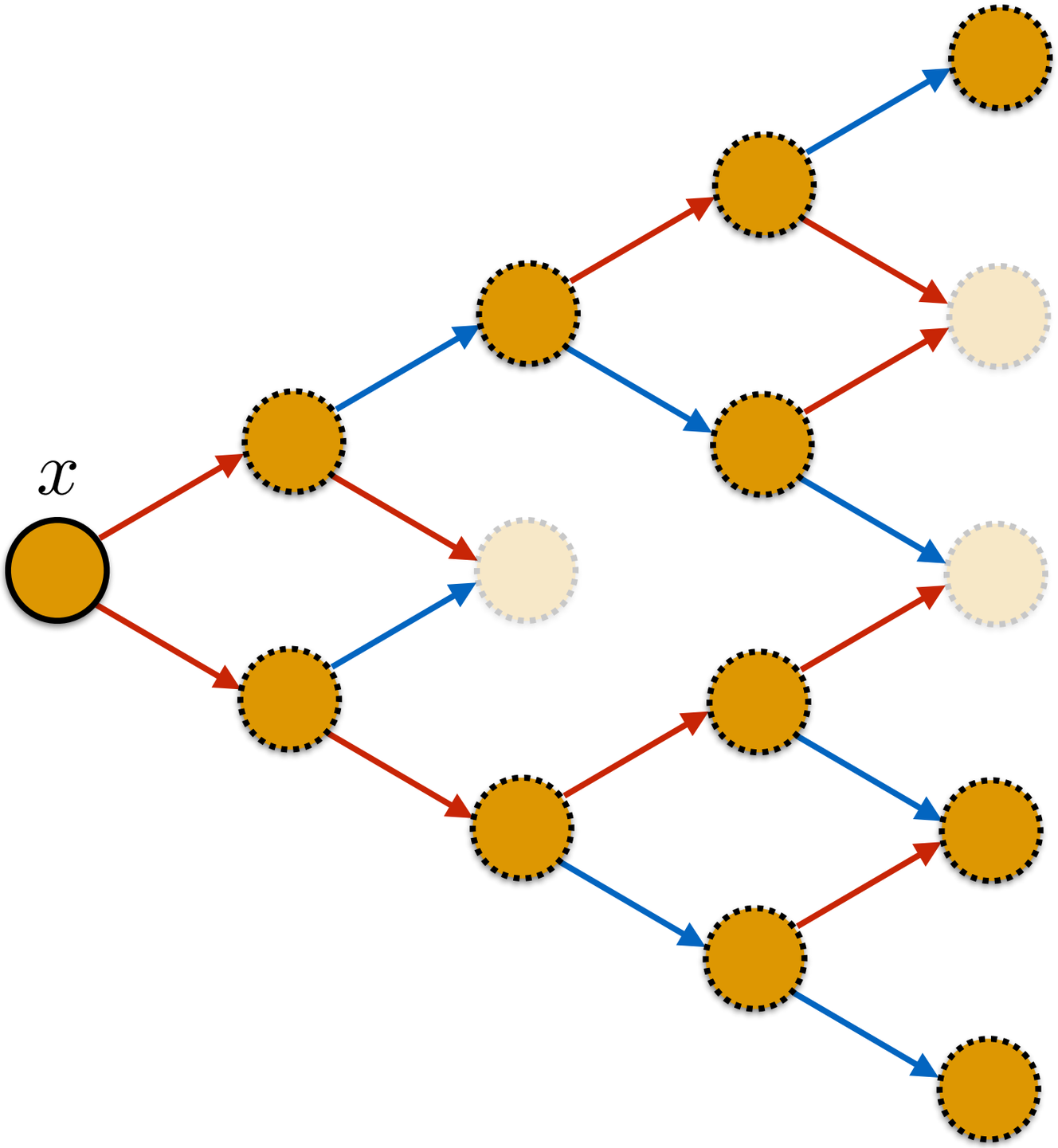

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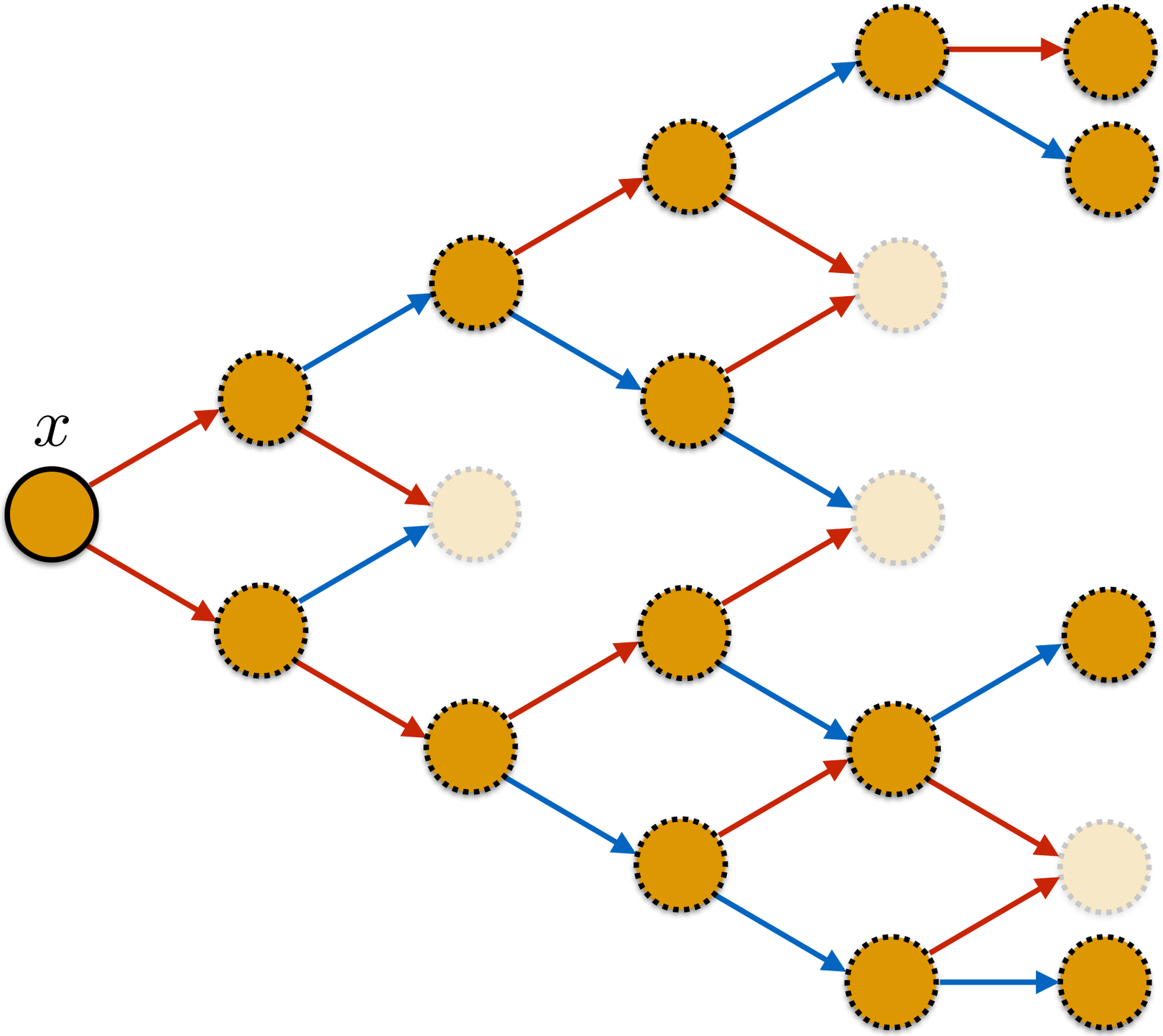

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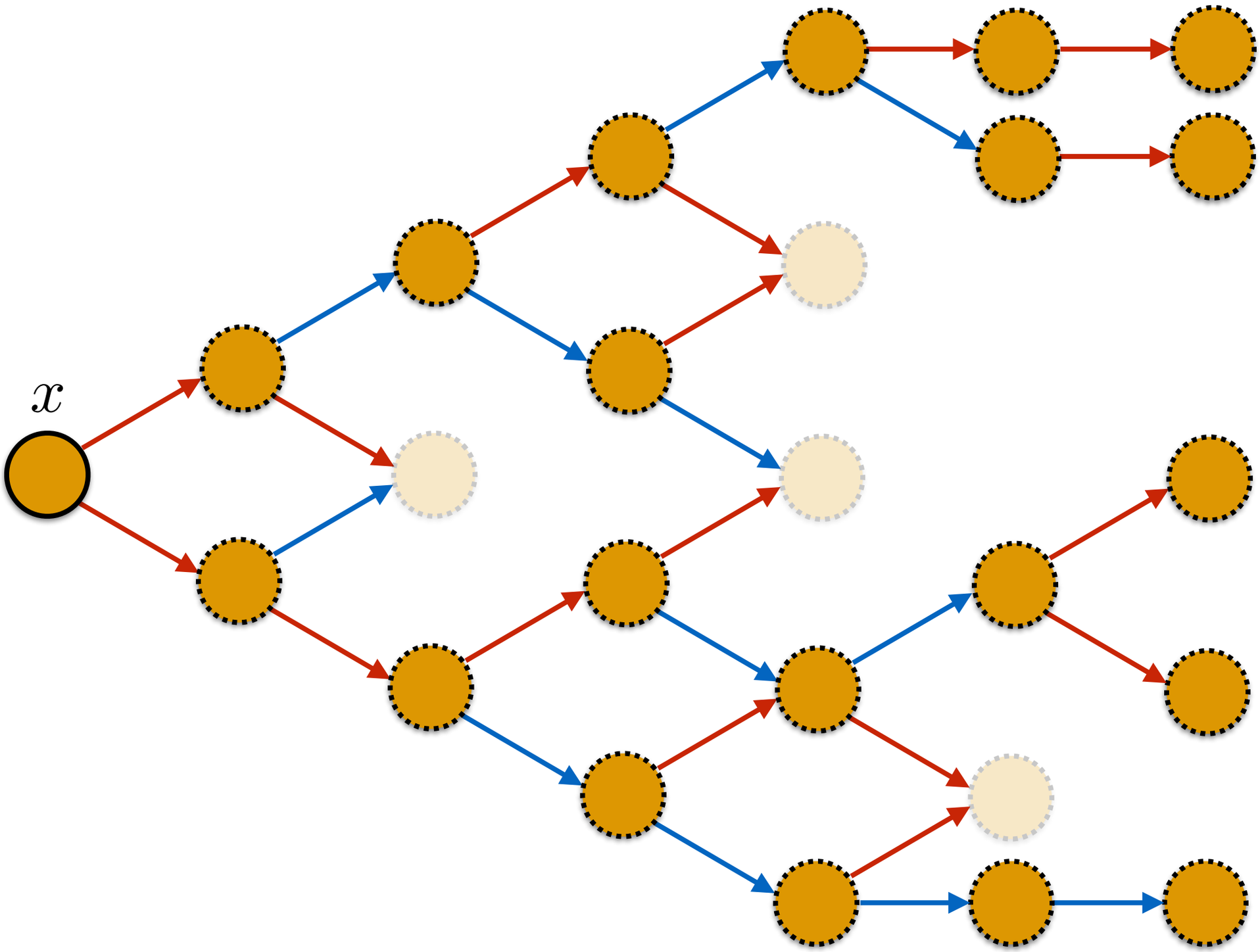

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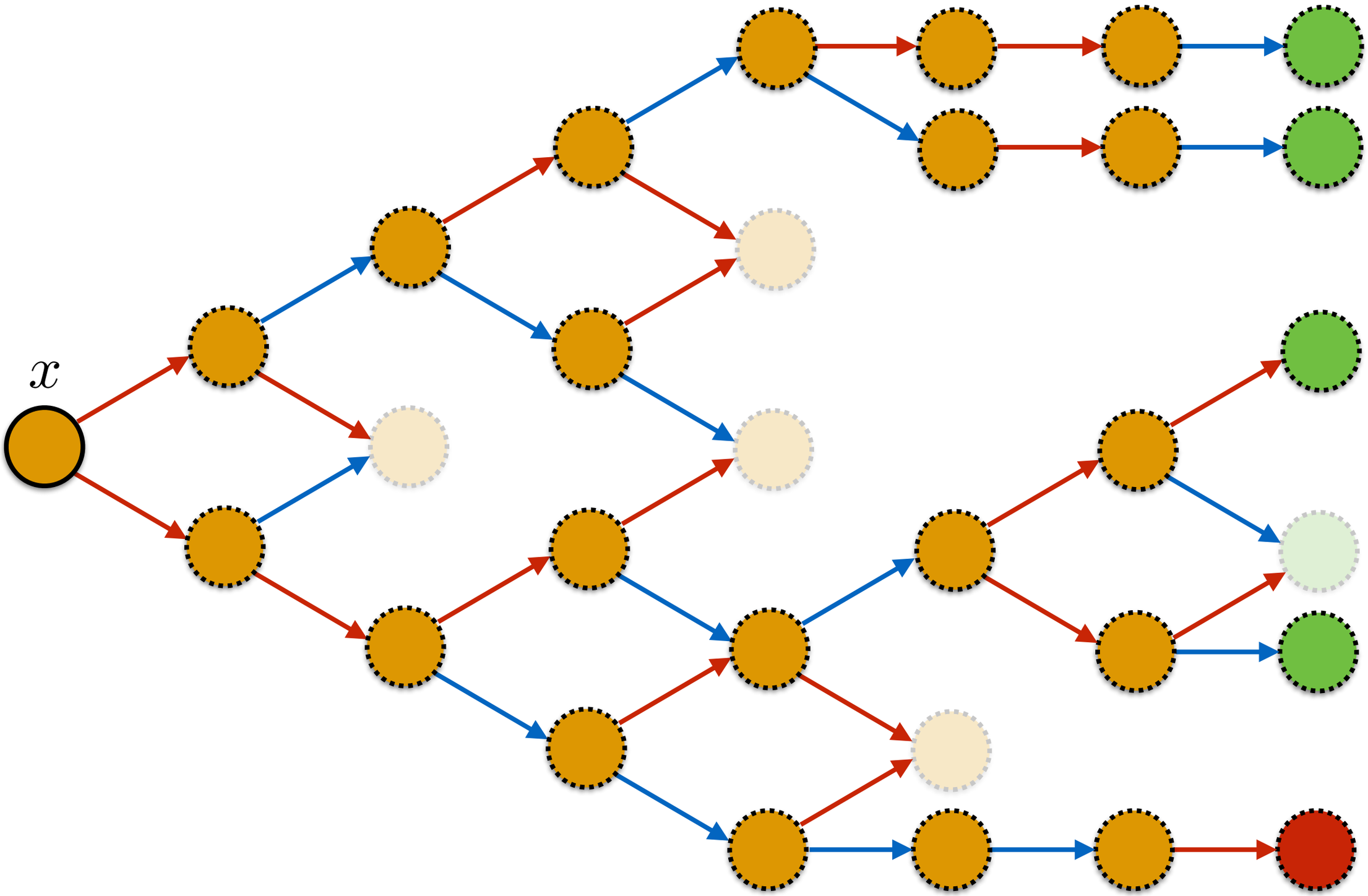

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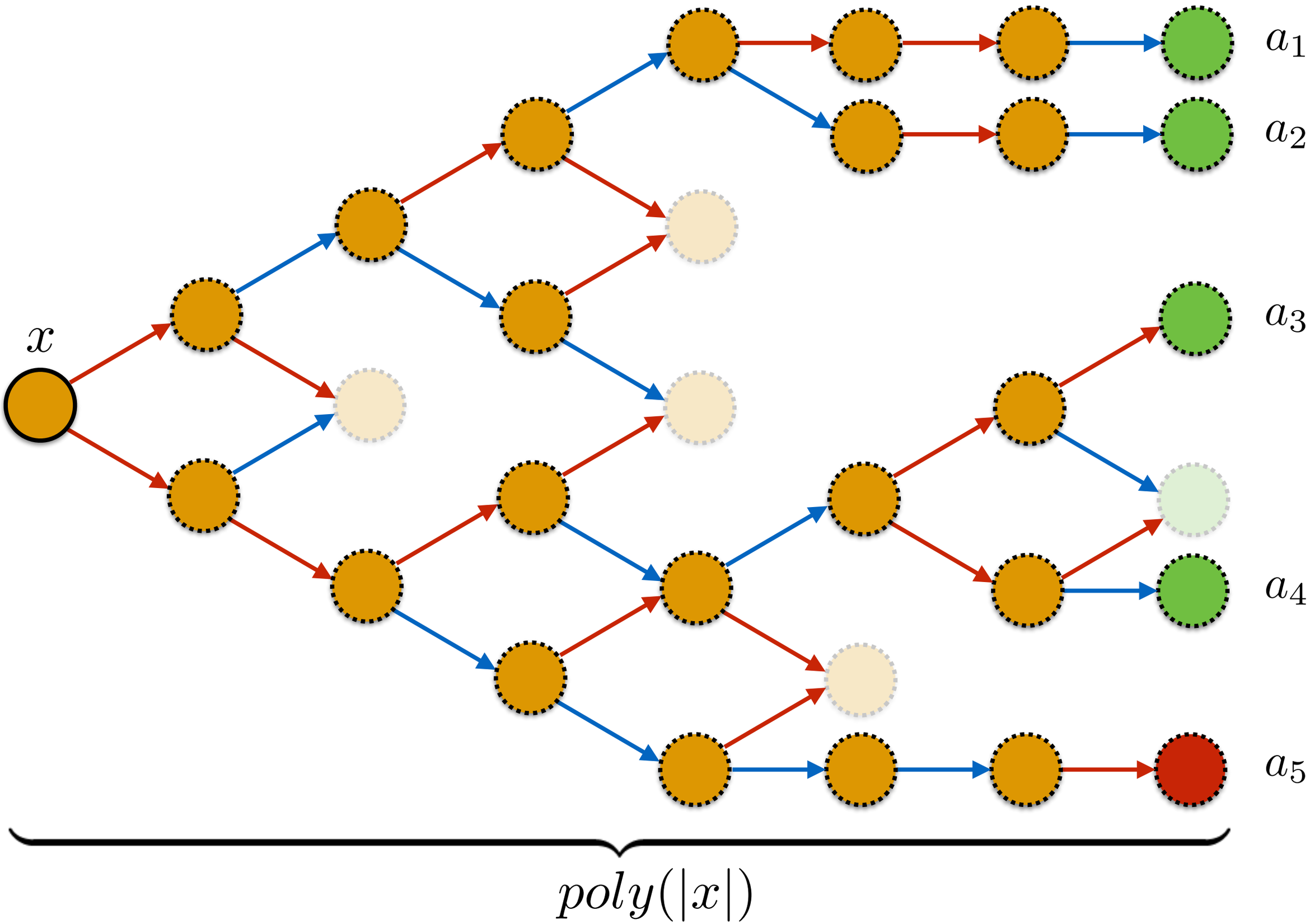
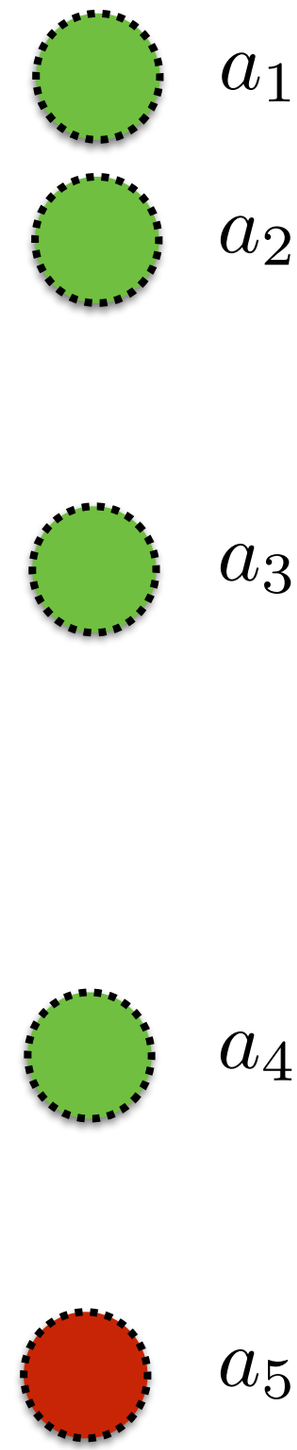
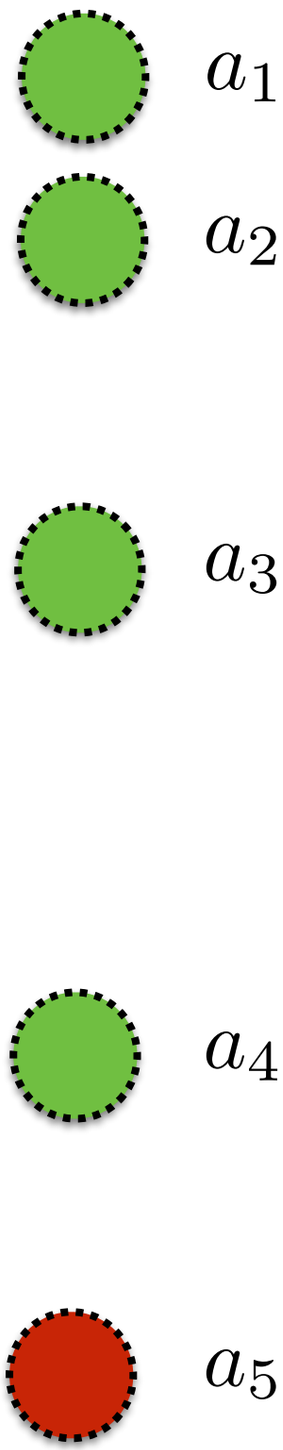


Quantum computation

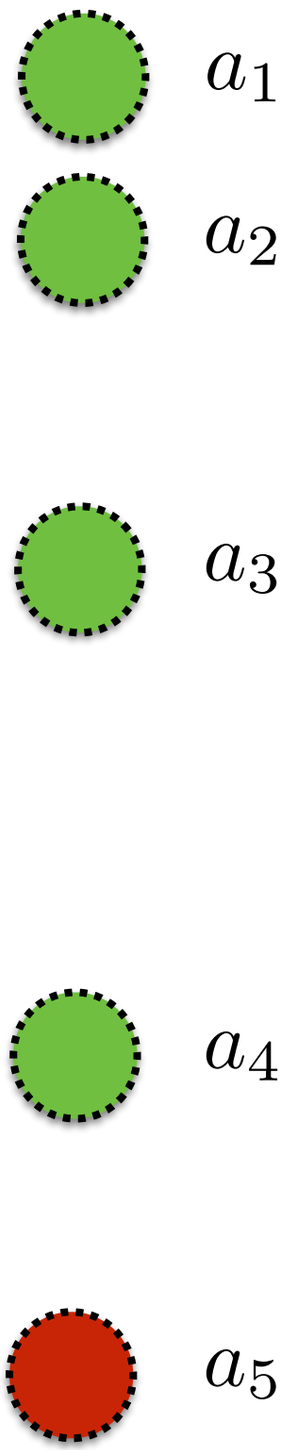


Quantum computation



$$Amp(YES) = a_1 + a_2 + a_3 + a_4$$

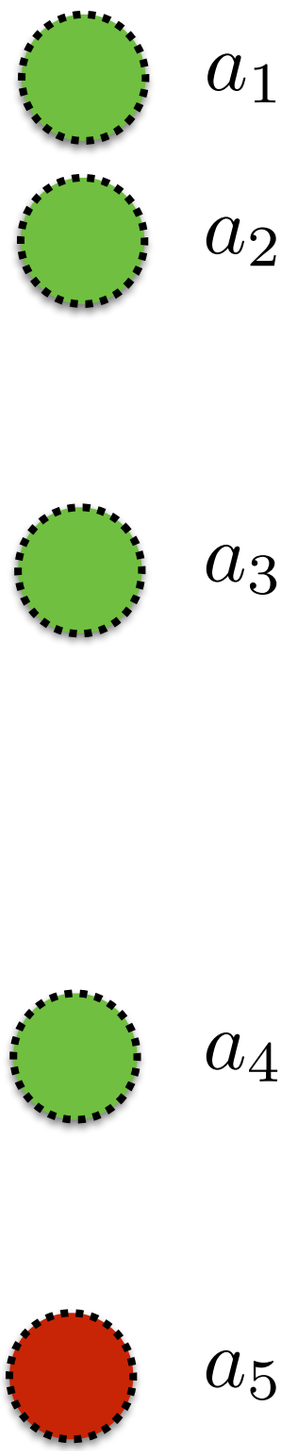
Quantum computation



$$Amp(YES) = a_1 + a_2 + a_3 + a_4$$

$$Pr(YES) = |a_1 + a_2 + a_3 + a_4|^2$$

Quantum computation

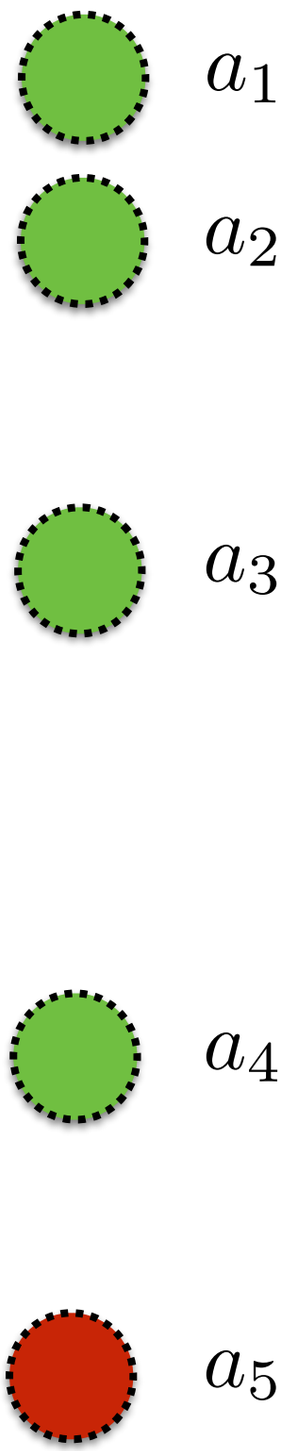


$$Amp(YES) = a_1 + a_2 + a_3 + a_4$$

$$Pr(YES) = |a_1 + a_2 + a_3 + a_4|^2$$

$$Pr(YES) = \left| \sum_{path \in YES-Paths} a_{path} \right|^2$$

Quantum computation



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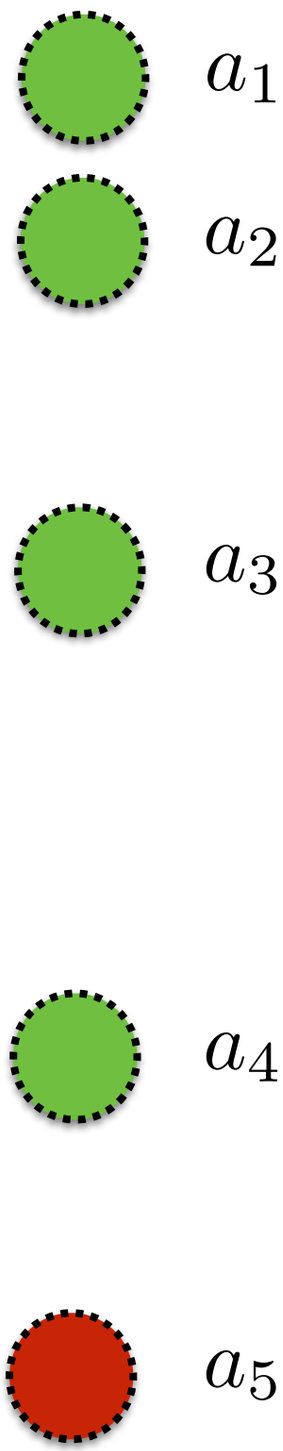
$$Pr(NO) = \left| \sum_{path \in NO-Paths} a_{path} \right|^2$$

Quantum computation



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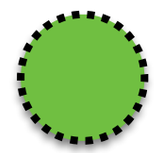
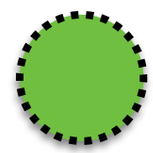
$$Pr(NO) = \left| \sum_{path \in NO-Paths} a_{path} \right|^2$$

$$Pr(YES) + Pr(NO) = 1$$

Quantum computation



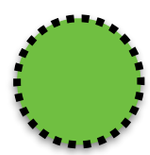
a_1



a_2

$$Amp(YES) = a_1 + a_2 + a_3 + a_4$$

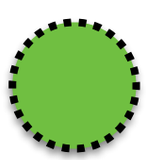
$$Pr(YES) = |a_1 + a_2 + a_3 + a_4|^2$$



a_3

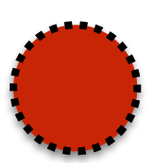
$$Pr(YES) = \left| \sum_{path \in YES-Paths} a_{path} \right|^2$$

$$Pr(NO) = \left| \sum_{path \in NO-Paths} a_{path} \right|^2$$



a_4

$$Pr(YES) + Pr(NO) = 1$$



a_5

When answer is YES, Pr(YES) > 2/3

When answer is NO, Pr(YES) < 1/3

Quantum computation

For simplicity, assume all amplitudes are real numbers

Quantum computation

For simplicity, assume all amplitudes are real numbers

$$\left(\sum_i a_i\right)^2 = \sum_i a_i^2 + \sum_{i \neq j} a_i a_j$$

Quantum computation

For simplicity, assume all amplitudes are real numbers

$$\left(\sum_i a_i\right)^2 = \sum_i a_i^2 + \boxed{\sum_{i \neq j} a_i a_j}$$

interference term

Quantum computation

For simplicity, assume all amplitudes are real numbers

$$\left(\sum_i a_i\right)^2 = \sum_i a_i^2 + \boxed{\sum_{i \neq j} a_i a_j}$$

interference term

Interference term close to 0 → classical computation

Quantum computation

For simplicity, assume all amplitudes are real numbers

$$\left(\sum_i a_i\right)^2 = \sum_i a_i^2 + \boxed{\sum_{i \neq j} a_i a_j}$$

interference term

Interference term close to 0 → classical computation

For everyday phenomena, amplitudes are fairly random

Quantum computation

For simplicity, assume all amplitudes are real numbers

$$\left(\sum_i a_i\right)^2 = \sum_i a_i^2 + \boxed{\sum_{i \neq j} a_i a_j}$$

interference term

Interference term close to 0 → classical computation

For everyday phenomena, amplitudes are fairly random

Interference term is close to 0

Quantum computation

For simplicity, assume all amplitudes are real numbers

$$\left(\sum_i a_i\right)^2 = \sum_i a_i^2 + \underbrace{\boxed{\sum_{i \neq j} a_i a_j}}_{\text{interference term}}$$

Interference term close to 0 → classical computation

For everyday phenomena, amplitudes are fairly random

Interference term is close to 0

This is why we don't see quantum weirdness around us!

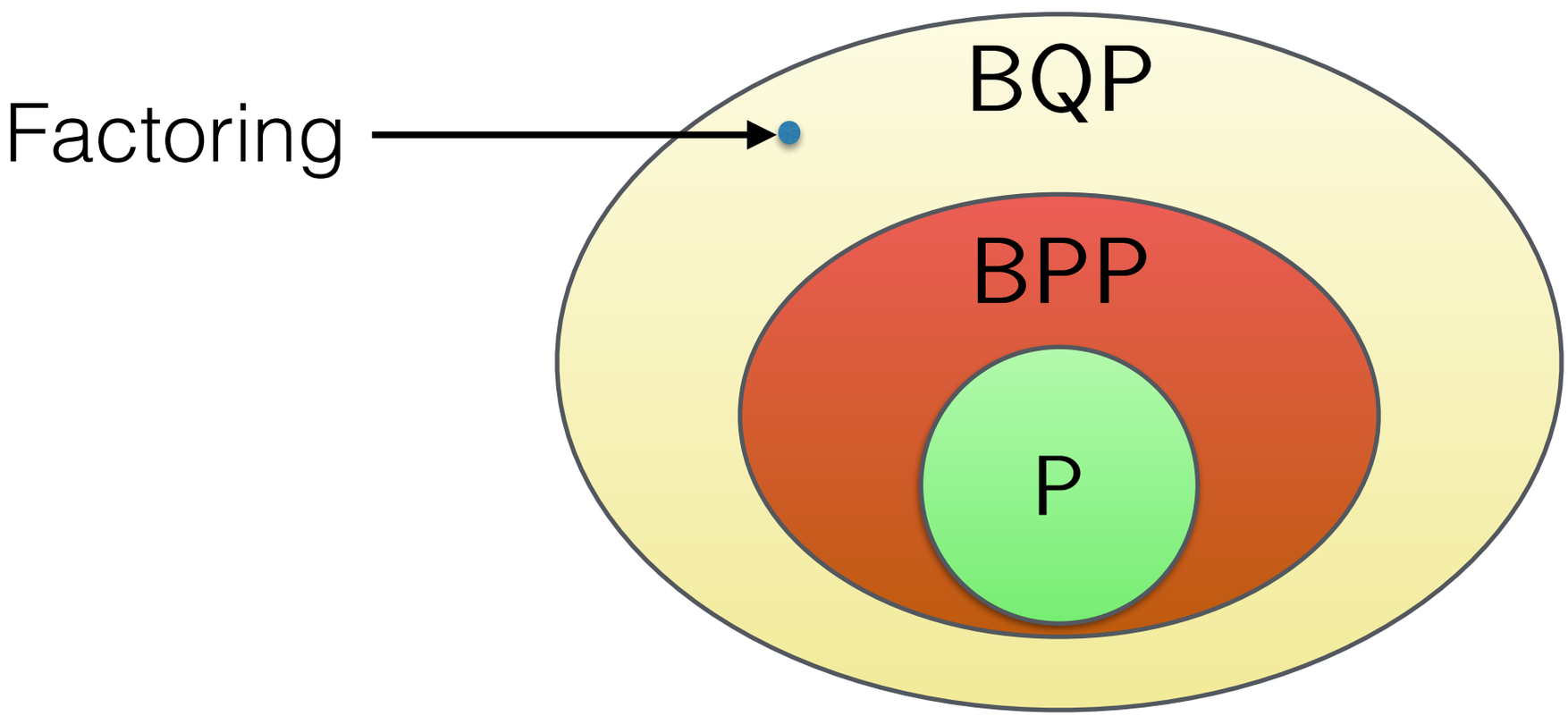
Classes of problems

Sets of problems that can be solved in polynomial time

Deterministic computation: **P**

Probabilistic computation: **BPP**

Quantum computation: **BQP**



Interference computation

Pick your programming language of choice

Imagine that in that language you had the function:

$$interf(f :: Set \to \{-1, +1\}, D :: Set) \to Bool$$

YES $$\frac{1}{|D|}\left|\sum_{x \in D} f(x)\right| \geq 2/3$$

NO $$\frac{1}{|D|}\left|\sum_{x \in D} f(x)\right| \leq 1/3$$

Otherwise, output is random

Interference computation

$$interf(f :: Set \to \{-1, +1\}, D :: Set) \to Bool$$

D can be specified **compactly**

E.g. $D = \{0, 1\}^n$

Would require exponential space to store D

f specified as pointer to some function

What can we solve with *interf*?

Deutsch-Josza problem

Suppose you can query the following function

$$f :: \{0,1\}^n \rightarrow \{0,1\}$$

The function is either **constant** or **balanced**

Constant: same value for all inputs

Balanced: 0 for half of inputs, 1 for other half

Constant functions:

$$\forall x \in \{0,1\}^n \; f(x) = 0 \qquad \forall x \in \{0,1\}^n \; f(x) = 1$$

Some balanced functions:

$$f(x) = parity(x)$$

$$f(x) = x_1$$

Deutsch-Josza problem

Suppose you can query the following function

$$f :: \{0,1\}^n \to \{0,1\}$$

The function is either **constant** or **balanced**

How many queries to determine type of function?

$$2^{n-1} + 1$$

Query function on half of domain + one extra element

What if we can use *interf*?

Deutsch-Josza problem

Let us define the following function:

$$g :: \{0,1\}^n \to \{-1,+1\} \quad g(x) = (-1)^{f(x)}$$

Call $interf(g, \{0,1\}^n)$

When f is constant

$$2^{-n} \left| \sum_{x \in \{0,1\}^n} g(x) \right| = 1 \to \mathbf{YES}$$

When f is balanced

$$2^{-n} \left| \sum_{x \in \{0,1\}^n} g(x) \right| = 0 \to \mathbf{NO}$$

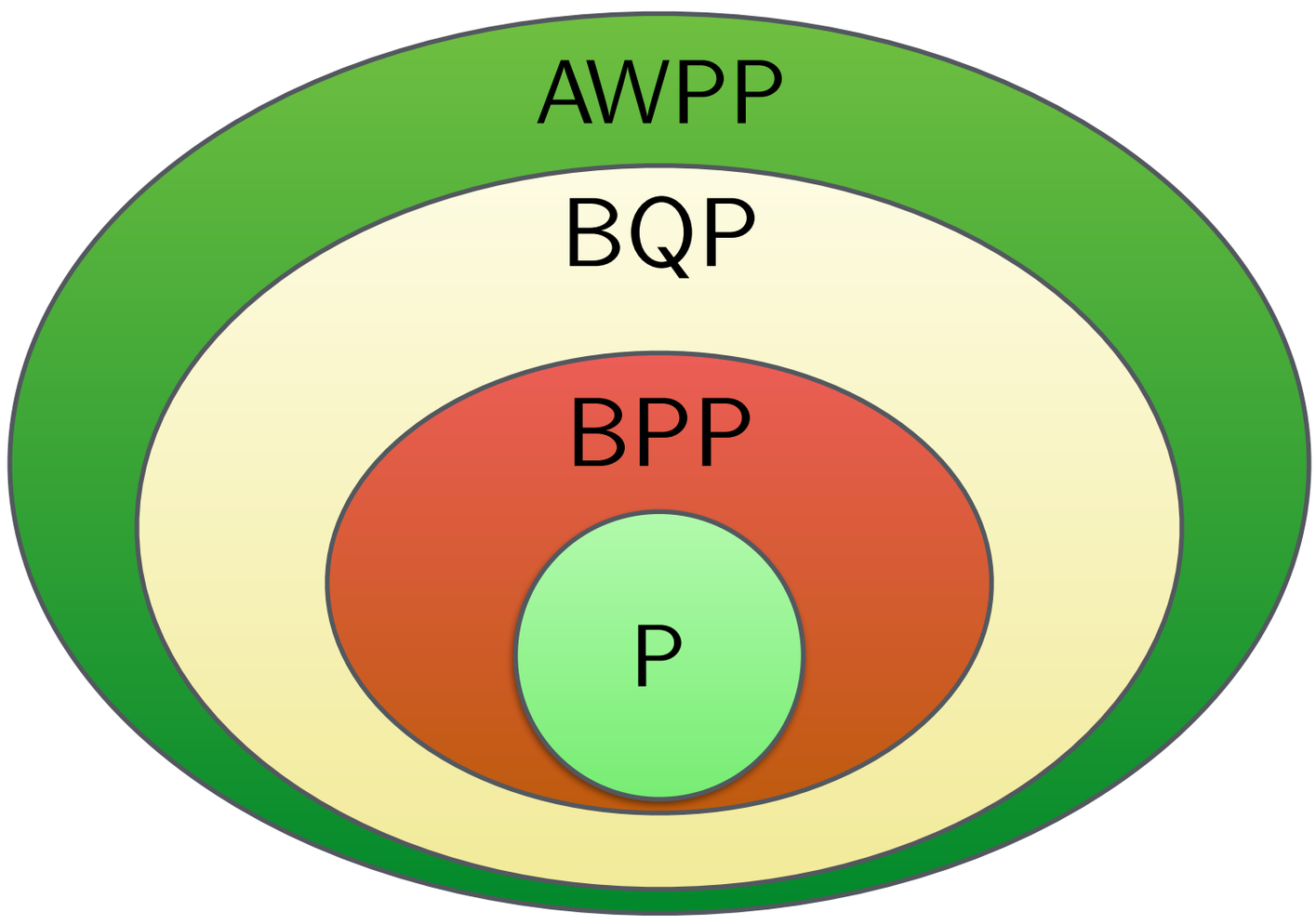
Interference computation

Quantum algorithms are mostly about implementing

$$interf(f :: Set \to \{-1, +1\}, D :: Set) \to Bool$$

Does classical computation + interf = quantum computation?

Probably not



The complexity zoo

QMA

coNP

NC

P/poly

NEXP

QCMA

ACC

L

PH

ONP

AM

BQP/qpoly

QIP

PostBPP

IP

MA

MIP

SZK

#P

PP

PostBQP

HVQSZK

QMA(2)

Favourite complexity class

YQP

Yoda Quantum Polynomial-time

https://arxiv.org/pdf/quant-ph/0608142.pdf

References and resources

Complexity Zoo
https://complexityzoo.uwaterloo.ca/Complexity_Zoo

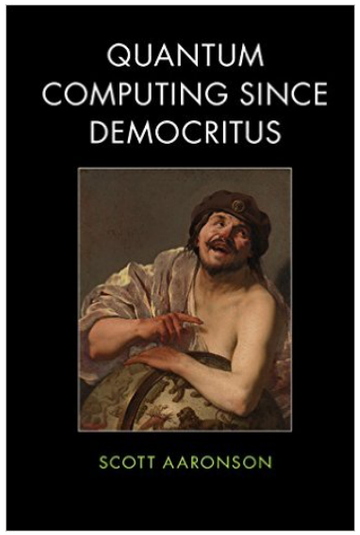
Complexity theory and quantum computation
http://stellar.mit.edu/S/course/6/fa14/6.845/materials.html

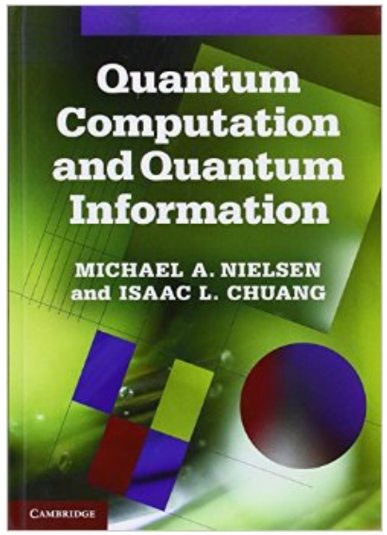
Complexity theory textbook
http://theory.cs.princeton.edu/complexity/book.pdf

Another great textbook
https://www.amazon.co.uk/Computational-Complexity-Christos-H-Papadimitriou/dp/0201530821

References and resources

For everything else…





References and resources

Images on slides 2,3,4,10

https://upload.wikimedia.org/wikipedia/commons/7/7e/Double-slit_experiment_results_Tanamura_2.jpg

https://upload.wikimedia.org/wikipedia/commons/thumb/8/86/Ebohr1_IP.svg/2000px-Ebohr1_IP.svg.png

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References and resources

Images on slides 12,13,25,79

http://www.thesoftwarecomplex.com/wp-content/uploads/2015/05/Runcomputerfaster.jpg

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https://vignette.wikia.nocookie.net/starwars/images/d/d6/Yoda_SWSB.png/revision/latest?cb=20150206140125

https://vecto.rs/600/vector-of-a-cartoon-happy-businesswoman-flipping-a-coin-coloring-page-outline-by-toonaday-20324.jpg