Quantum Computation and Cryptography
Day 8

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Part II
Where are the quantum computers?

Physical implementations

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So what's the problem? Why can't we scale these implementations?

The problem

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Everything else

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Everything else ... i.e. **the environment**

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How?

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We say that the qubit has lost coherence, or decohered

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While decoherence is damaging to computation, it explains why we don't see quantum weirdness around us

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Let's look at a different question first

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This leads to our solution...

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- Computation is run with the encoded qubits

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Syndrome measurement $\approx$ measurement which gives us some information about the (logical) state without disturbing it

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This is called **syndrome measurement**
(or Quantum Non-Destructive measurement, QND)

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This can be generalized to detect any kind of error
(on a single physical qubit), with 9 physical qubits!

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Examples of assumptions regarding errors (per interval):

- Environment independently corrupts a qubit with probability p_1
- Single qubit gates fail with some probability p_2
- Two-qubit gates fail with some probability p_3 (correlated error)

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E.g. to have small error rate per interval, make circuit such that at most one gate acts per qubit (in an interval)

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Quantum memories would also have to do this!

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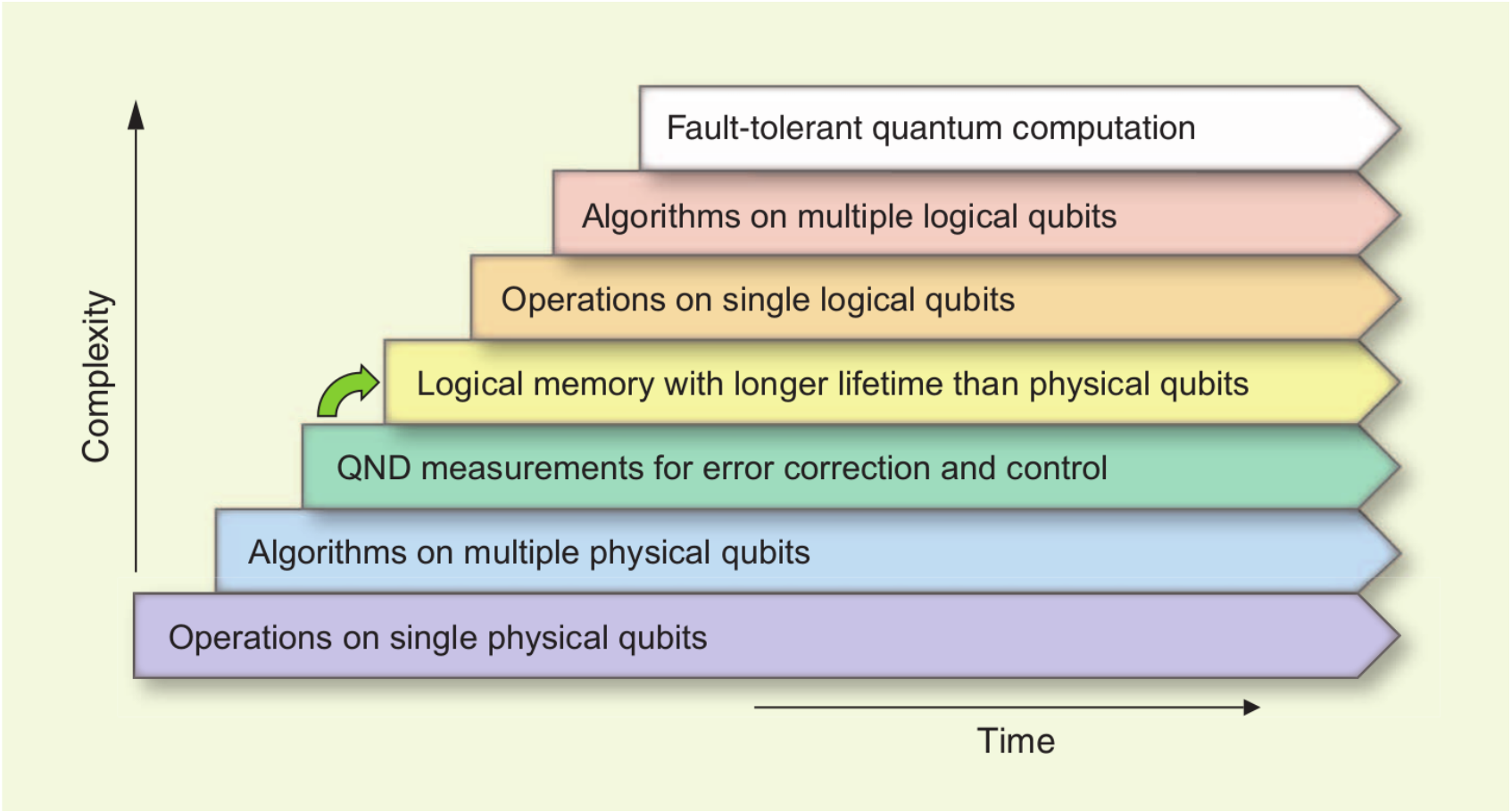
- They are also trying to make it scalable
- Only a technological challenge, not a physical one!
- D-Wave machines are not universal, nor fault tolerant

The current picture



Useful resources

- **The physical implementation of quantum computing** - http://www-users.cs.york.ac.uk/schmuel/book/quantum.pdf
- **NQIT and Q20:20** - http://nqit.ox.ac.uk/
- **Linear Optics Quantum Computation: an Overview** - http://arxiv.org/pdf/quant-ph/0512104v1.pdf
- **Superconducting Circuits for Quantum Information: An Outlook** (also image on slide 18) - http://www.sciencemag.org/content/339/6124/1169.full
- **Against the scepticism of quantum computing** - http://www.scottaaronson.com/democritus/lec14.html
- **Decoherence and hidden variables** - http://www.scottaaronson.com/democritus/lec11.html
- **Decoherence explained mathematically** - Section 11 in http://arxiv.org/pdf/quant-ph/0011013.pdf
- **The role of decoherence in quantum mechanics** - http://plato.stanford.edu/entries/qm-decoherence/

Useful resources

- **Quantum Error Correction for Beginners** - http://arxiv.org/pdf/0905.2794v4.pdf
- **An Introduction to Quantum Error Correction and Fault-Tolerant Quantum Computation** - http://arxiv.org/abs/0904.2557
- **Stabilizer codes and Quantum Error Correction** - http://arxiv.org/pdf/quant-ph/9705052v1.pdf
- **Magic states** - http://www.iqst.ca/events/csqic05/talks/nathan%20b.pdf