Quantum Computation and Cryptography
Day 6

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Part I
Quantum entanglement

Brief recap

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The properties that can be observed about a quantum system are called observables. They correspond to Hermitian operators. The observed values are the eigenvalues of these operators, and the system "collapses" to the corresponding eigenstate.

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The observable determines the basis!

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Notice that X and Z are not diagonal in the same basis!

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What happens if we measure the first qubit?

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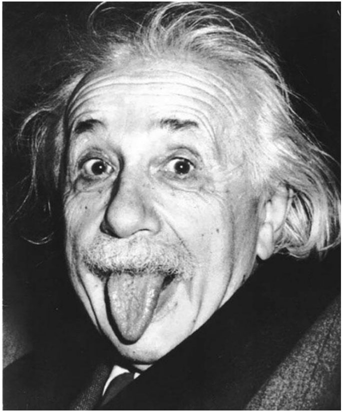

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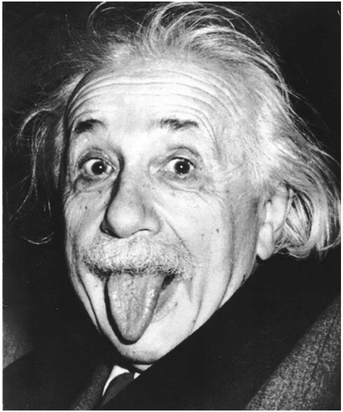

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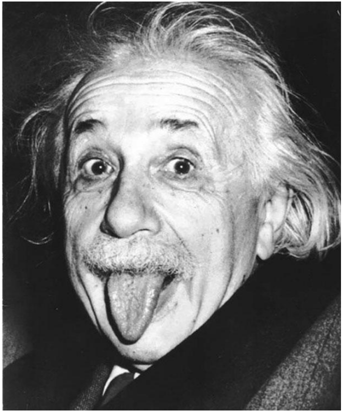

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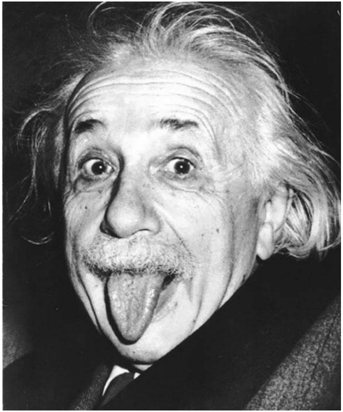
- Together with Podolsky and Rosen they came up with a "paradox"



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Einstein wasn't "married" to determinism,
but he was "married" to locality so he chose option 1

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Einstein wasn't "married" to determinism,
but he was "married" to locality so he chose option 1

"*God does not play dice*"

Resolution and Bell's theorem

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Can we test for locality?

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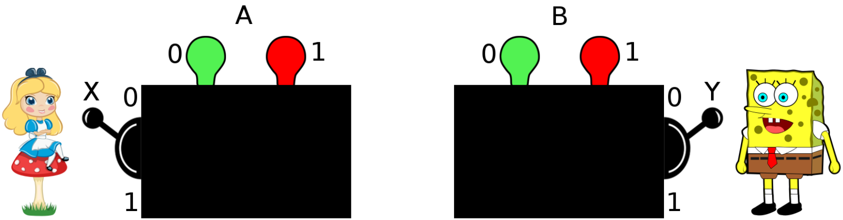

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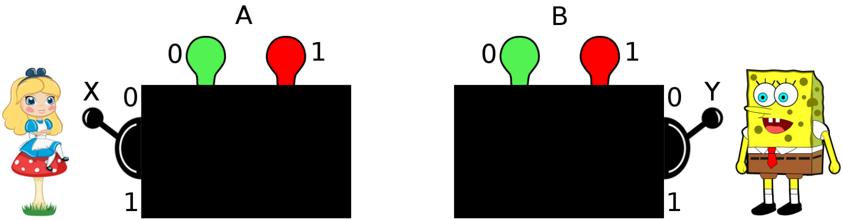

Locality:

$$Pr(A,B|X,Y) = \sum_{\lambda} Pr(A|X,\lambda)Pr(B|Y,\lambda)Pr(\lambda)$$

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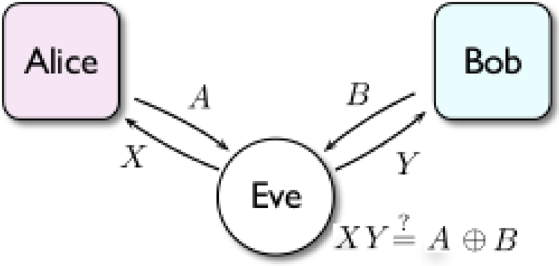

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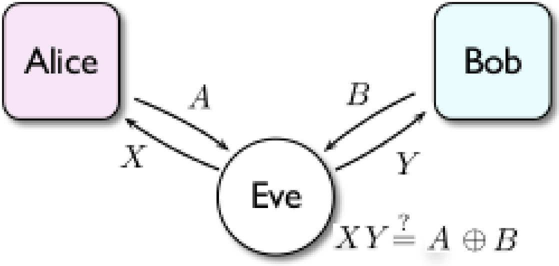

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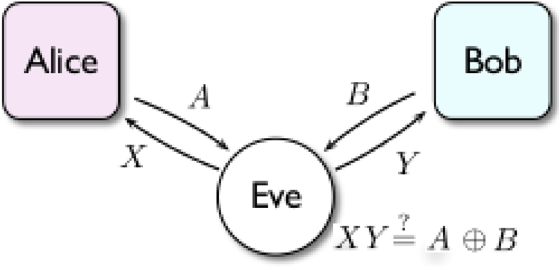

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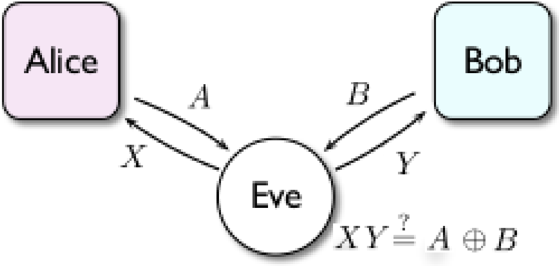

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It can be shown (a form of Bell's theorem):
$Pr(XY = A \oplus B) \leq 75\%$

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Max win rate equivalent to (CHSH inequality):

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We can test this!

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Resolution and Bell's theorem

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$$\langle \Phi^+ | \frac{1}{\sqrt{2}}(X \otimes X + X \otimes Z) | \Phi^+ \rangle$$
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Resolution and Bell's theorem

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And so we get:

$$E(A'B'|0,0)+E(A'B'|0,1)+E(A'B'|1,0)-E(A'B'|1,1) = 2\sqrt{2} > 2$$

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Last year (2015) was the first loophole-free Bell inequality test

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- Non-locality is actually about correlations

Non-locality vs entanglement

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- Not all entangled states produce non-local correlations
- One can quantify entanglement based on this fact
- The $|\Phi^+\rangle$ Bell state is maximally entangled
- Observing non-local correlations $\rightarrow$ entanglement
- Quantum correlations are not the "strongest" though (Popescu-Rohlich correlations)

Bell states

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One more nice application of entanglement

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So any $|\chi\rangle = a\,|\Phi^+\rangle + b\,|\Phi^-\rangle + c\,|\Psi^+\rangle + d\,|\Psi^-\rangle$

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What is $|00\rangle$ in this basis?

$$|00\rangle = (|\Phi^+\rangle + |\Phi^-\rangle)/\sqrt{2}$$

Being a basis... we can measure two qubits in this basis!

Application

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$$(|0\rangle_H |0\rangle_G + |1\rangle_H |1\rangle_G)/\sqrt{2}$$

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$$|\psi\rangle = a|0\rangle + b|1\rangle$$

Application

$$(|0\rangle_H |0\rangle_G + |1\rangle_H |1\rangle_G)/\sqrt{2}$$

$$|\psi\rangle = a\,|0\rangle + b\,|1\rangle$$

What if Hansel measures his 2 qubits in the Bell basis?

We'd have to write Hansel's qubits in the Bell basis...

Application

$$(a\,|0\rangle_H + b\,|1\rangle_H) \otimes (|0\rangle_H\,|0\rangle_G + |1\rangle_H\,|1\rangle_G)/\sqrt{2}$$

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$$\ldots$$

$$\frac{1}{\sqrt{2}}\left|\Phi^+\right\rangle_H (a\,|0\rangle_G + b\,|1\rangle_G) + \frac{1}{\sqrt{2}}\left|\Phi^-\right\rangle_H (a\,|0\rangle_G - b\,|1\rangle_G) +$$

$$\frac{1}{\sqrt{2}}\left|\Psi^+\right\rangle_H (b\,|0\rangle_G + a\,|1\rangle_G) + \frac{1}{\sqrt{2}}\left|\Psi^-\right\rangle_H (b\,|0\rangle_G - a\,|1\rangle_G)$$

Application

$$(a|0\rangle_H + b|1\rangle_H) \otimes (|0\rangle_H|0\rangle_G + |1\rangle_H|1\rangle_G)/\sqrt{2}$$

$$(a|00\rangle_H|0\rangle_G + a|01\rangle_H|1\rangle_G + b|10\rangle_H|0\rangle_G + b|11\rangle_H|1\rangle_G)/\sqrt{2}$$

$$\ldots$$

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If Hansel measures and the outcome is $|\Phi^+\rangle$ what's Gretel's qubit?

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Quantum teleportation

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Original was destroyed and recreated on the other side!
Would you be teleported?

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Would you be teleported?

Teleportation only possible with entanglement!
(no teleportation theorem)

Useful resources

- **Commutation $\iff$ simultaneous measurement** - http://ocw.mit.edu/courses/physics/8-04-quantum-physics-i-spring-2013/study-materials/MIT8_04S13_OnCommEigenbas.pdf
- **What Bell did** - http://arxiv.org/pdf/1408.1826v1.pdf
- **EPR paper** - http://www.drchinese.com/David/EPR.pdf
- **Non-locality beyond quantum mechanics** - http://www.nature.com/nphys/journal/v10/n4/full/nphys2916.html
- **Quantum computing and hidden variables** - http://www.scottaaronson.com/papers/qchvpra.pdf
- **Loophole free Bell inequality test** - http://www.nature.com/nature/journal/v526/n7575/full/nature15759.html

Useful resources

- **Better explanation for magic square game** - https://users.wpi.edu/~paravind/Publications/MSQUARE5.pdf
- **Bohmiam mechanics and non-local hidden variables** - http://plato.stanford.edu/entries/qm-bohm
- **Can the quantum state be interpreted statistically?** - http://mattleifer.info/2011/11/20/can-the-quantum-state-be-interpreted-statistically/
- **Teleporting quantum gates** - https://www.perimeterinstitute.ca/personal/dgottesman/teleportgates.html
- **Quantum psuedo-telepathy** - http://arxiv.org/pdf/quant-ph/0407221.pdf
- **Quantum bidding in bridge** - http://journals.aps.org/prx/abstract/10.1103/PhysRevX.4.021047
- **The cost of exactly simulating quantum entanglement with classical communication** - http://arxiv.org/pdf/quant-ph/9901035v1.pdf

References

- Image on slide 11 (Einstein) - http://images.mentalfloss.com/sites/default/files/styles/insert_main_wide_image/public/einstein1_7.jpg
- Image on slide 14 (CHSH game) - Ben Reichardt, Falk Unger, Umesh Vazirani, A classical leash for a quantum system: Command of quantum systems via rigidity of CHSH games http://arxiv.org/abs/1209.0448
- Image on slide 19 (Bell) - http://www.quotationof.com/images/john-s-bell-1.jpg

All other images are open clipart