

# Quantum Computation and Cryptography

## Day 10

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## Part II

### Fun stuff

# Quantum gravity

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(one candidate for this is *string theory*)

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Inside there's a singularity, a very strange thing  
(relativity predicts infinite curvature of space-time at singularity)

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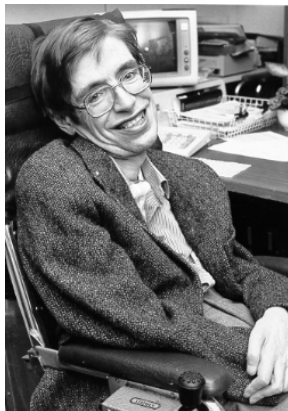
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Which in turn led to a striking realization...

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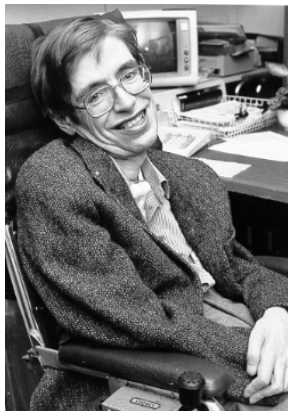
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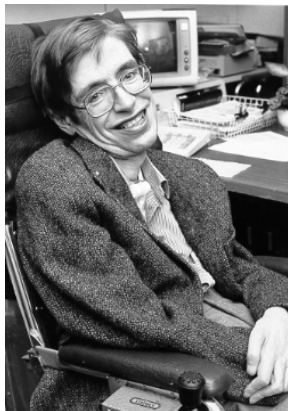
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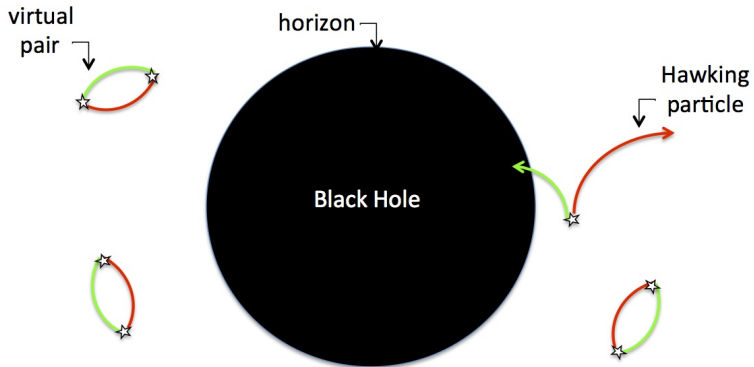
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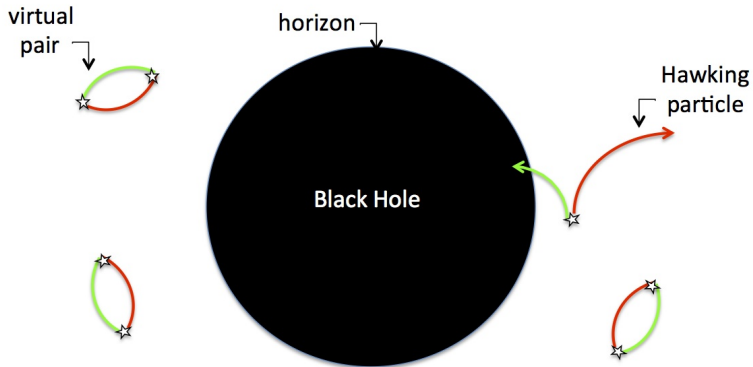
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We'll call it **theory X**

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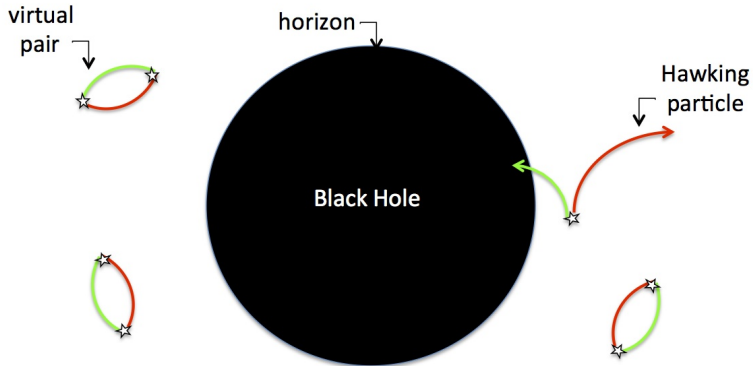


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Can take a really really reaaaaally long time to evaporate  
( $10^{67}$  years for black hole with mass of the Sun)



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That means that *outgoing radiation* should be correlated with  
*infalling matter*

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Moreover theory X is kind of self-contradictory since it wants to be a quantum theory (and so should have unitary evolution)

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“So physicists came up with a potential solution called *black hole complementarity*. I didn't really understand it. Fortunately for me, years later the physicists realized they don't understand it either.”

**Scott Aaronson**

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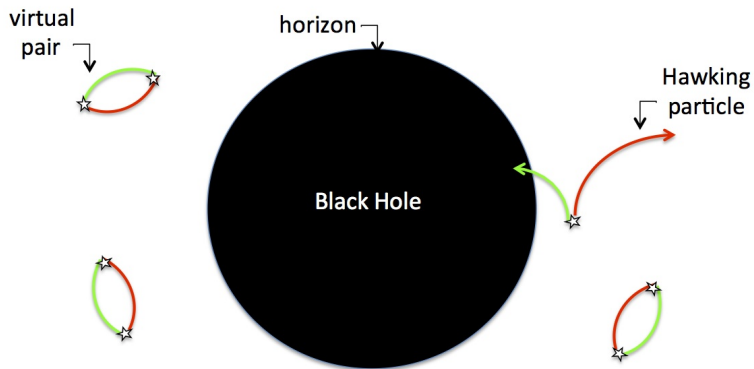
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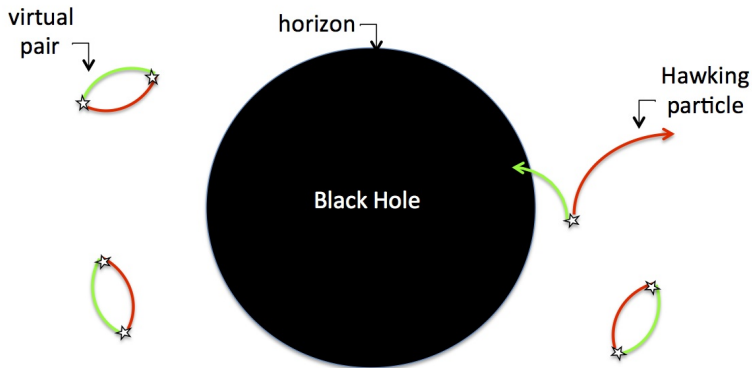
This means that there is a *firewall* at the event horizon

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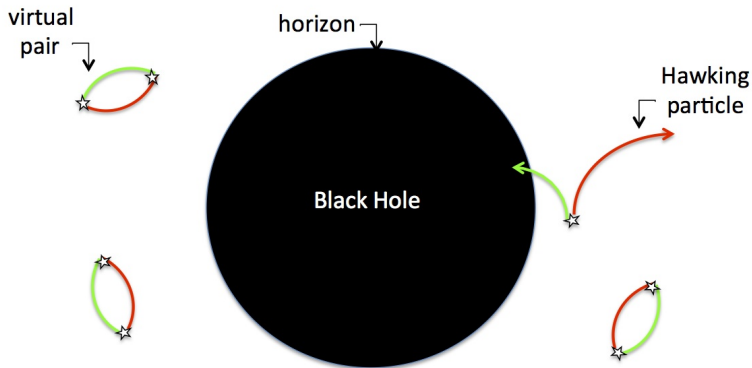


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So an emerging radiation qubit is maximally entangled with one inside the black hole



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Such that:  $(U' \otimes I)|\phi\rangle = |\Phi_+\rangle \otimes |\text{rest}\rangle$

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The last radiation qubit to have emerge after  $T_P$  is maximally entangled with one of the other  $k - 1$  radiation qubits

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$|\text{particles}\rangle \rightarrow |\text{black hole}\rangle \rightarrow |\text{final hawking radiation}\rangle$

The evolution is unitary

Examine the state after **over half** of the black hole has evaporated  
(the time for this to happen is called Page time  $T_P$ )

We have:  $|\psi(T_P)\rangle = U|0\rangle^{\otimes N}$

Wlog the first  $k = N/2 + ct$  are the radiation qubits

The last radiation qubit to have emerge after  $T_P$  is maximally entangled with one of the other  $k - 1$  radiation qubits

But EFT + no drama  $\rightarrow$  it is also maximally entangled with one of the  $N - k$  qubits

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This contradicts **monogamy of entanglement!**

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AMPS: she will hit a firewall when she tries to go inside

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## Result 1 (Harlow-Hayden):

If you can solve HH in quantum poly time, then you can solve another problem believed to be difficult (*set equality*) in quantum poly time

## Result 2 (Aaronson):

If you can solve HH in quantum poly time, then you can invert any one-way function (OWF) in quantum-poly time

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This is just an interesting perspective

Can't really say much more without a *quantum theory of gravity*

Briefly mentioning some other unexpected applications of quantum information...

# Quantum computation without quantum computers

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(inspired from [*Drucker and de Wolf '09*])



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Same idea with the formalism of quantum computation/crypto

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And others...

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But all of them require **quantum RAM**



# Quantum finance

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Behold the Black-Scholes equation:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0$$

$V$  - price of an option

$S$  - stock price

$t$  - time

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Hamiltonian for a charged particle in an EM field

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E.g. quantum binomial options pricing model (see references)

These fall into the area of *econophysics*

# And many other quantum things

Quantum chemistry

Quantum logic

Quantum game theory

Categorical quantum mechanics (and computation)

Quantum cognitive science

(just using the mathematics of quantum mechanics, nothing to do  
with quantum consciousnesses)

Etc

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*“If computers that you build are quantum  
Then spies everywhere will all want'em  
Our codes will all fail,  
And they'll read our email,  
Till we get crypto that's quantum, and daunt'em”*  
Jennifer and Peter Shor

## Useful resources and references

- **Black hole thermodynamics** - [https://en.wikipedia.org/wiki/Black\\_hole\\_thermodynamics](https://en.wikipedia.org/wiki/Black_hole_thermodynamics)
- **Hawking radiation** - [https://en.wikipedia.org/wiki/Hawking\\_radiation](https://en.wikipedia.org/wiki/Hawking_radiation)
- **Black hole information paradox** - [https://en.wikipedia.org/wiki/Black\\_hole\\_information\\_paradox](https://en.wikipedia.org/wiki/Black_hole_information_paradox)
- **Hawking-Thorne-Preskill bet** - [https://en.wikipedia.org/wiki/Thorne%E2%80%9393Hawking%E2%80%9393Preskill\\_bet](https://en.wikipedia.org/wiki/Thorne%E2%80%9393Hawking%E2%80%9393Preskill_bet)
- **A complexity perspective on black holes** - <http://www.scottaaronson.com/barbados-2016.pdf>
- **Black hole complementarity and firewalls** - <https://arxiv.org/pdf/1301.4505v2.pdf>
- **Awesome summer school (with video lectures) about quantum gravity and quantum information** - <https://www.perimeterinstitute.ca/conferences/it-qubit-summer-school>

## Useful resources and references

- **AMPS paper** - <https://arxiv.org/abs/1207.3123>
- **Hayden and Harlow paper** -  
<https://arxiv.org/abs/1301.4504>
- **Sean Carroll explaining firewall problem** -  
[https://www.youtube.com/watch?v=\\_8bhtEgB8Mo](https://www.youtube.com/watch?v=_8bhtEgB8Mo)
- **Scott Aaronson explaining decoding problem** -  
<https://www.youtube.com/watch?v=zJdTtL3ajaI>
- **Black holes as mirrors** -  
<https://arxiv.org/abs/0708.4025>
- **Quantum algorithms for machine learning** -  
<https://arxiv.org/abs/1409.3097>
- **Quantum algorithms for solving linear systems** -  
<https://arxiv.org/abs/0811.3171>
- **Quantum machine learning: read the fine print** -  
<http://www.scottaaronson.com/papers/qml.pdf>
- **Quantum finance** -  
[https://en.wikipedia.org/wiki/Quantum\\_finance](https://en.wikipedia.org/wiki/Quantum_finance)

## Useful resources and references

- **Quantum computation without quantum computers -**  
<http://arxiv.org/abs/0910.3376>
- **Quantum Black-Scholes -**  
<https://arxiv.org/abs/0706.1300v1>
- **Quantum methods for option pricing -**  
<https://arxiv.org/abs/cond-mat/0005319v1>
- **Quantum chemistry -**  
[https://en.wikipedia.org/wiki/Quantum\\_chemistry](https://en.wikipedia.org/wiki/Quantum_chemistry)
- **Quantum chemistry on quantum computer -**  
<http://www.nature.com/nchem/journal/v2/n2/abs/nchem.483.html>
- **Quantum logic -**  
<http://plato.stanford.edu/entries/qt-quantlog/>

## Useful resources and references

- **Quantum game theory** - [https://en.wikipedia.org/wiki/Quantum\\_game\\_theory](https://en.wikipedia.org/wiki/Quantum_game_theory)
- **Quantum voting and quantum dictatorships** - <https://arxiv.org/abs/1501.00458>
- **Categorical quantum mechanics** - [https://en.wikipedia.org/wiki/Categorical\\_quantum\\_mechanics](https://en.wikipedia.org/wiki/Categorical_quantum_mechanics)
- **An introduction to categorical quantum mechanics** - <http://arxiv.org/abs/1510.05468>



# References

- Image on slide 4 (black hole from Interstellar) - <https://i.ytimg.com/vi/YdSz12G1hlw/maxresdefault.jpg>
- Image on slide 7 (Stephen Hawking) - [https://upload.wikimedia.org/wikipedia/commons/e/eb/Stephen\\_Hawking.StarChild.jpg](https://upload.wikimedia.org/wikipedia/commons/e/eb/Stephen_Hawking.StarChild.jpg)
- Image on slides 8 and 13 (virtual particles black hole) - <http://physics.princeton.edu/~verlinde/hawkingpair.jpg>